Exchange and CPT

Daniel L. Miller

Compare SO(3N,N) with N × SO(3,1)
- Embedding
- CPT
- Exchange

Commutations between CPT and Exchange
- SO(3N,N)
- N × SO(3,1)

Conclusions
- matter–antimatter asymmetry
- Call for experiments
- Few notes

Proof of anticommutation between exchange and charge conjugation for spinors

Interactions between identical Particles (Matter) and between identical Anti-Particles (Anti-Matter) are not the same.

Daniel L. Miller

Daniel.Miller@Intel.Com

APS April meeting 2021

Compare SO(3N,N) with $N \times SO(3,1)$
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
$N \times SO(3,1)$
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

Outline

This work is based on various group embeddings published in 1990–2010 by prof. M. Berry and other teams, and author works from 2014-2016; preprint was 1st submitted in 2019

1. Compare SO(3N,N) with $N \times SO(3,1)$
   - Embedding
   - CPT
   - Exchange
2. Commutations between CPT and Exchange
   - SO(3N,N)
   - $N \times SO(3,1)$
3. Conclusions
   - matter–antimatter asymmetry
   - Call for experiments
   - Few notes

Compare SO(3N,N) with N×SO(3,1) – Embedding

Exchange and CPT
Daniel L. Miller
Compare SO(3N,N) with N × SO(3,1)
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
N × SO(3,1)
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

N×SO(3,1) world can be embedded into SO(3N,N); It gives rise to spinor branching rules; we will not need them explicitely; just compare γ-matrices.

SO(3N,N)

Bispinor has 2^{2N} components
Vector has $4N$ components

$$\gamma_j^\mu = \underbrace{\gamma^{\text{FIVE}} \otimes \cdots}_{j-1\text{ times}} \gamma^\mu \underbrace{\cdots \otimes \hat{1}}_{N-j\text{ times}}$$

$$\{\gamma_j^\mu, \gamma_k^\nu\} = 2\delta_{jk}\delta_{\mu\nu}$$

N×SO(3,1)

N×Bispinor has 2^{2N} components
N Vectors have $4N$ components

$$\gamma_j^\mu = \underbrace{\hat{1} \otimes \cdots}_{j-1\text{ times}} \gamma^\mu \underbrace{\cdots \otimes \hat{1}}_{N-j\text{ times}}$$

$$[\gamma_j^\mu, \gamma_k^\nu] = 0,\ j \neq k$$

$$\{\gamma_j^\mu, \gamma_j^\nu\} = 2\delta_{\mu\nu}$$

γ-matrices are different in their commutation realtions, but dimensions and amounts are the same.

Compare SO(3N,N) with N×SO(3,1) – CPT

Exchange and CPT
Daniel L. Miller
Compare SO(3N,N) with N × SO(3,1)
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
N × SO(3,1)
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

CPT transformations are defined in a similar way

$$\hat{C} = \prod_j \hat{C}_j,\ \hat{C}_j \sim \gamma_j^2 \qquad \hat{P} = \prod_j \hat{P}_j,\ \hat{P}_j \sim \gamma_j^0 \tag{1}$$

$$\hat{T} = \prod_j \hat{T}_j,\ \hat{T}_j \sim \gamma_j^1 \gamma_j^3 \qquad \hat{I}^4 = \prod_j \hat{I}_j^4,\ \hat{I}_j^4 \sim \gamma_j^0 \gamma_j^1 \gamma_j^2 \gamma_j^3$$

$$\hat{I}^4 = \hat{C}\hat{P}\hat{T}$$

Compare SO(3N,N) with N × SO(3,1)
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
N × SO(3,1)
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

SO(3N,N) vs N×SO(3,1) – Exchange

Exchange is $2^{2N} \times 2^{2N}$ matrix operator acting on bi-spinor and its coordinates.

It exchanges components of coordinates of these bi-spinors in the same way for SO(3N,N) and N×SO(3,1)

$$\hat{E}_{ij} : \begin{array}{ll} x_j^\mu \to x_i^\mu \,, & x_i^\mu \to x_j^\mu \,, \\ x_k^\mu \to x_k^\mu \,, & k \neq i, j \end{array} \qquad (2)$$

SO(3N,N)

The Exchange operator can be derived explicitly; properties

$$\hat{E}_{ij}^x \, \gamma_j^\mu \, \hat{E}_{ij}^x = \gamma_i^\mu \qquad (3)$$

$$\hat{E}_{ij}^x = \hat{E}_{ji}^x \quad \hat{E}_{ij}^x \hat{E}_{ij}^x = \hat{1}$$

N×SO(3,1)

The Exchange operator E^h was postulated in 1926 by Werner Heisenberg for non-relativistic quantum mechanics. It can be generalized for QED

Commutations between C, P, T, CPT and Exchange

Exchange and CPT
Daniel L. Miller
Compare SO(3N,N) with N × SO(3,1)
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
N × SO(3,1)
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

The anticommutation of the γ-matrices between themselves, along with Eqs. (1,3), gives for SO(3N,N)

$$\begin{aligned} \hat{E}^x \hat{C} \hat{E}^x &= -\hat{C} & \qquad \hat{E}^x \hat{P} \hat{E}^x &= -\hat{P} \\ \hat{E}^x \hat{T} \hat{E}^x &= \hat{T} & \qquad \hat{E}^x \hat{I}^4 \hat{E}^x &= \hat{I}^4 \end{aligned} \tag{4}$$

For N$\times$SO(3,1) Exchange commutes with all CPT symmetries, this is must for proof of the the spin-statistics theorem.

$$\begin{aligned} \hat{E}^h \hat{C} \hat{E}^h &= \hat{C} & \qquad \hat{E}^h \hat{P} \hat{E}^h &= \hat{P} \\ \hat{E}^h \hat{T} \hat{E}^h &= \hat{T} & \qquad \hat{E}^h \hat{I}^4 \hat{E}^h &= \hat{I}^4 \end{aligned} \tag{5}$$

Symmetry lowering SO(3N,N) $\to$ N$\times$SO(3,1) and the proof of anticommutation between exchange and charge conjugation

Compare SO(3N,N) with N$\times$SO(3,1)
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
N$\times$SO(3,1)
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

Central hypothesis

- Let's postulate our word was initially 4N-dimensional, SO(3N,N)
- Then the Big Bang reduced the spatial symmetry to SO(3,1), giving rise to N spinor fields, namely SO(3,1) bi-spinors
- Let's assume the Exchange operator $\hat{E}$ and all many body CPT transformations are preserved upon the symmetry lowering

$$E = E^x \Rightarrow \ \hat{E}\hat{C}\hat{E} = -\hat{C}, \quad \{\hat{E}, \hat{C}\} = 0$$

- This ends the proof of anticommutation between exchange and charge conjugation

Consequences for matter–antimatter asymmetry

Exchange and CPT

Daniel L. Miller

Compare SO(3N,N) with N $\times$ SO(3,1)

Embedding

CPT

Exchange

Commutations between CPT and Exchange

SO(3N,N)

N $\times$ SO(3,1)

Conclusions

matter–antimatter asymmetry

Call for experiments

Few notes

$$\{\hat{E}, \hat{C}\} = 0 \quad \{\hat{E}, \hat{P}\} = 0 \quad [\hat{E}, \hat{T}] = 0$$

- the charge conjugation reverses the statistics (for spinors)
- bosonic fields annihilate fermionic fields
- no changes in single line diagrams (eg magnetic moment)
- no Periodic Table for antimatter; collapse of antiatom size $Z^{-1/3} \to Z^{-1}$ due to lack of the degeneracy pressure
- this explains matter–antimatter asymmetry within QED
- it would be no selfconjugated (truly neutral) spinors
- vector fields stay bosonic (contract matter with matter)
- scalar contracting matter with antimatter must be the supercharge

Call for experiments

Exchange and CPT

Daniel L. Miller

Compare SO(3N,N) with N × SO(3,1)
Embedding
CPT
Exchange
Commutations between CPT and Exchange
SO(3N,N)
N × SO(3,1)
Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

Few experiments can either confirm or invalidate anticommutation between exchange and charge conjugation. As of today none of such evidences is available.

Possible evidences:

- spectroscopy of antihydrogen molecule
- positron–positron Møller scattering
- compare internal degeneracy of proton and anti–proton

Compare SO(3N,N) with N $\times$ SO(3,1)
Embedding
CPT
Exchange

Commutations between CPT and Exchange
SO(3N,N)
N $\times$ SO(3,1)

Conclusions
matter–antimatter asymmetry
Call for experiments
Few notes

Lorentz invariance

The Exchange operator derived by making use of rotations in SO(3N,N) is Lorentz invariant in SO(3,1)

Many body CPT

Many body CPT transformations Eq. (1) as they defined for SO(3N,N) can be used for N$\times$SO(3,1). So the assumption that CPT transformations are preserved upon the symmetry lowering is merely semantical. The assumption that the Exchange is preserved upon the symmetry lowering is critical.

Explicit Exchange matrices: New one

$$\hat{E}_{12} = \begin{pmatrix}
1 & & & & & & & & & & & & & & & \\
 & & & & 1 & & & & & & & & & & & \\
 & & & & & & & & 1 & & & & & & & \\
 & & & & & & & & & & & & 1 & & & \\
 & 1 & & & & & & & & & & & & & & \\
 & & & & & -1 & & & & & & & & & & \\
 & & & & & & & & & -1 & & & & & & \\
 & & & & & & & & & & & & & 1 & & \\
 & & 1 & & & & & & & & & & & & & \\
 & & & & & & -1 & & & & & & & & & \\
 & & & & & & & & & & -1 & & & & & \\
 & & & & & & & & & & & & & & 1 & \\
 & & & 1 & & & & & & & & & & & & \\
 & & & & & & & 1 & & & & & & & & \\
 & & & & & & & & & & & 1 & & & & \\
 & & & & & & & & & & & & & & & 1
\end{pmatrix}$$

Explicit Exchange matrices: Old one

Compare SO(3N,N) with N × SO(3,1)

Embedding

CPT

Exchange

Commutations between CPT and Exchange

SO(3N,N)

N × SO(3,1)

Conclusions

matter–antimatter asymmetry

Call for experiments

Few notes

$$\hat{E}_{\mathrm{WH}} = -\begin{pmatrix}
1 &&&&&&&&&&&&&&& \\
&&&& 1 &&&&&&&&&&& \\
&&&&&&&& 1 &&&&&&& \\
&&&&&&&&&&&& 1 &&& \\
& 1 &&&&&&&&&&&&&& \\
&&&&& 1 &&&&&&&&&& \\
&&&&&&&&& 1 &&&&&& \\
&&&&&&&&&&&&& 1 && \\
&& 1 &&&&&&&&&&&&& \\
&&&&&& 1 &&&&&&&&& \\
&&&&&&&&&& 1 &&&&& \\
&&&&&&&&&&&&&& 1 & \\
&&& 1 &&&&&&&&&&&& \\
&&&&&&& 1 &&&&&&&& \\
&&&&&&&&&&& 1 &&&& \\
&&&&&&&&&&&&&&& 1
\end{pmatrix}$$