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The Spontaneous breaking of the Continuous Particle Exchange Symmetry

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The paper reports the commuting Right Weil spinors and the anticommuting Left Weil spinors obtained by the spontaneous symmetry breaking of the $4N$ dimensional massless Lagrangian to 4 dimensions. Once $4N-4$ dimensions and related fields are integrated out, the statistics flip operator (scalar supercharge) emerges. The restrictions of the Coleman-Mandula theorem are automatically fulfilled. In the same time the scalar supercharge allows to bypass the Pauli principle for the Right Weil spinor.

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I. INTRODUCTION

The continuous particle exchange was introduced by Berry and Robbins[1, 2] to realize mathematically the belt trick[3], and demonstrate the Pauli principle in $SO(3)$ non-relativistic quantum mechanics. Many detailed reviews were published since first Pauli paper[4] dealing with relation between spin and statistics.[5–7] Let me also mention the recent interest to the foundations of the Pauli principle.[8, 9] All cited papers converge to same results as stated by this theorem and discuss in details its connection to the $\hat{C}\hat{P}\hat{T}$-invariance, topology, symmetry of the scattering process, locality etc.

This paper reports the Lorentz-invariant and $\hat{C}\hat{P}\hat{T}$-invariant field theory with the anticommuting left spinors and the commuting right spinors. The supersymmetry and the $\hat{C}\hat{P}\hat{T}$-invariance are combined by making use of the statistics flip operator (scalar supercharge) Q. It allows to go around the Pauli principle while preserving the positively defined energy and the odd current upon the 4-inversion ($\hat{C}\hat{P}\hat{T}$ transformation).

The text book exposition[10] of the Pauli principle makes use of the non-invariant particle and anti-particle creation and annihilation operators, while the original Pauli's paper was formulated in the Lorentz-invariant form. The former approach is used in the next paper[11] where we show that for the present theory the $\hat{C}\hat{P}\hat{T}$ transformation swaps both the charge and the statistics of particles. The theory has the positively defined energy and the correctly defined current thus meeting all conditions of the spin-statistics connection theorem.

This paper calculates the exchange of $SO(1,3)$ spinors in two steps. We start (Sec. II) with a $4N$-dimensional space and calculate the transformation of a $SO(N,3N)$ bi-spinor upon simultaneous $\pi/2$ rotations of four planes. In this way we define the subspace exchange matrix. The next and last step (Sec. III) is to break the symmetry to $SO(1,3)$ and assume that the subspace exchange matrix is the true particle permutation operator.

The original expectation was to get a sign flip of all 2^{2N} components of the field upon exchange of two 4-dimensional subspaces. It turns out to be not the case. The main result of this work is that the symmetry breaking of $SO(N,3N)$ to $SO(1,3)$ leaves the anticommuting left spinors and the commuting right spinors. Besides, once the $4N-4$ spacial degrees of freedom and corresponding fields are integrated out we stay with the boson-fermion swap operator (supercharge) in the Lagrangian. This operator allows to define the Lorentz-invariant supersymmetric Lagrangian in 4-dimensions.

The modern supersymmetric QED Lagrangians have supercharges in spinor representations[12, 13] in order to satisfy restrictions of both the Coleman-Mandula theorem and the Pauli principle. The present work come up with the scalar supercharge, which satisfies the Coleman-Mandula theorem and, as shown in the next paper[11], helps to bypass the Pauli principle.

The scalar supercharge was first introduced in the theory of disordered conductors.[14] That theory has fermionic and bosonic charges in the same spinor representation, describing the same physical particle, therefore it is the "single" particle theory. The Pauli principle plays no role there, except for keeping currents on the the Fermi surface.

Here the situation is different, there are bosonic and fermionic charges describing different particles, but both contribute to the anticommuting left and the commuting right fields. The connection between fields and charges is made by the super-rotation generated by Q.[11]

II. EXCHANGE OF 4-DIMENSIONAL SUBSPACES INSIDE A $4N$-DIMENSIONAL SPACE

The theory start with ψ^N and $\psi^{\dagger N}$ being field operators in $SO(N,3N)$ space; they transform as a 2^{2N} component bi-spinors of the D_{2N} Lie group

$$\psi^N = \psi^N_L + \psi^N_R \sim \left(\begin{array}{c} [0,0,\ldots,1,0]^N \\ [0,0,\ldots,0,1]^N \end{array} \right), \qquad (1)$$

and the same structure holds for $\psi^{\dagger N}$. Here the highest Dynkin weights in square brackets denote the $D(2N)$ left (also the subscript "L") and right spinors (also the subscript "R").[15] In what follows I'll omit upperscript index for $N=1$. The complex conjugation preserves the handedness for even N and reverses it for odd N. In this work we assume the odd N.

The motion of this field is governed by the $SO(N,3N)$ "Lorenz" invariant Lagrangian

$$\mathcal{L}_N = i\sum_{j=1}^{N}\sum_{\mu=0}^{3}\psi^{\dagger N}\gamma^{\text{ZERO}}\gamma_j^\mu\partial_{j\mu}\psi^N \qquad (2)$$

where $4N$ axises are counted by groups of four (subspaces). The index j counts groups, and the index μ counts axises within each group. Within each group we use the space metrics $(1,-1,-1,-1)$. The dimensionality of the field operator $[\psi^N]$ is $[x]^{\frac{2}{(4N-1)}}$ leading to the dimensionless action $\int d^{4N}x\mathcal{L}_N$.

Similarly to the case of $N=1$ the product of the left and the right spinors contains the vector $[1,0,\ldots,0]$

$$\begin{aligned}&[0,0,\ldots,1,0]*[0,0,\ldots,0,1]=[1,0,\ldots,0]\\&+[0,0,1,0\ldots,0]+\ldots+[0,0,\ldots,1,1]\end{aligned}$$

and the product of the two e.g. left spinors contains the scalar

$$\begin{aligned}&[0,0,\ldots,1,0]*[0,0,\ldots,1,0]=[0,0,\ldots,0]\\&+[0,1,0\ldots,0]+\ldots+[0,0,\ldots,2,0]\ .\end{aligned}$$

The Lagrangian Eq. 2 is the scalar product of the vector ∂_μ and the vector made from the product of the left and the right spinors.

The set of $4N$ anticommuting γ-matrices in Eq.(2) depends on N. In what follows we will assume that the dimension of the γ-matrices matches nearby ψ^N field. So the explicit way of writing γ-matrices is

$$\begin{aligned}\gamma_j^0 &= \underbrace{1\otimes\ldots\otimes 1}_{2(j-1)}\otimes\sigma_1\otimes\underbrace{\sigma_3\ldots\otimes\sigma_3}_{2(N-j)+1}\\ \gamma_j^3 &= i\underbrace{1\otimes\ldots\otimes 1}_{2(j-1)}\otimes\sigma_2\otimes\underbrace{\sigma_3\ldots\otimes\sigma_3}_{2(N-j)+1}\\ \gamma_j^2 &= i\underbrace{1\otimes\ldots\otimes 1}_{2(j-1)+1}\otimes\sigma_1\otimes\underbrace{\sigma_3\ldots\otimes\sigma_3}_{2(N-j)}\\ \gamma_j^1 &= -i\underbrace{1\otimes\ldots\otimes 1}_{2(j-1)+1}\otimes\sigma_2\otimes\underbrace{\sigma_3\ldots\otimes\sigma_3}_{2(N-j)}\end{aligned}$$

and few auxiliary matrices are

$$\begin{aligned}\gamma_j^{\text{FIVE}} &= -i\gamma_j^0\gamma_j^1\gamma_j^2\gamma_j^3\\ \gamma^{\text{FIVE}} &= \underbrace{\sigma_3\otimes\ldots\otimes\sigma_3}_{2N}\end{aligned}$$

where $\sigma_1,\sigma_2,\sigma_3$ are the Pauli matrices. Similarly to the case $N=1$, the matrices $\gamma_j^0,\gamma_j^1,\gamma_j^3$ are real and the matrices γ_j^2 are imaginary.

The space rotation in the plane $(j\mu, j'\mu')$ by angle θ is equivalent to the linear transformation of the $4N$ coordinates $x\to Rx$ and 2^{2N} components of the wave function

$$\psi^N(x) \;\to\; U\psi^N(R^{-1}x)\ , \quad U=e^{(\theta/2)\gamma_j^\mu\gamma_{j'}^{\mu'}} \qquad (3a)$$

$$\psi^{\dagger N}(x) \;\to\; \psi^{\dagger N}(R^{-1}x)U^\dagger\ . \qquad (3b)$$

The action of rotation operator R can be verified by

$$U\gamma_j^\mu x_{j\mu}U^{-1}=\gamma_j^\mu(Rx)_{j\mu}\ . \qquad (3c)$$

Finally with the definition

$$\gamma^{\text{ZERO}}\equiv\prod_{j=1}^{N}\gamma_j^0\ , \quad \gamma^{\text{ZERO}}U^{-1}\gamma^{\text{ZERO}}=U^\dagger\ . \qquad (3d)$$

one arrives at the $4N$ "Lorenz" invariant Lagrangian. In other words Eqs. (3) preserve the Lagrangian Eq. (2) upon rotations in $4N$-dimensional space.

There is a transformation, which I call the $i\leftrightarrow j$ subspace exchange

$$\forall\mu\ E_{ij}\gamma_i^\mu E_{ij}=\gamma_j^\mu\ , \quad E_{ij}\gamma_j^\mu E_{ij}=\gamma_i^\mu \qquad (4)$$

and our purpose is to calculate the field transformation by this subspace exchange

$$\psi^{N\prime}(\ldots x_i\ldots x_j\ldots)=E_{ij}\psi^N(\ldots x_j\ldots x_i\ldots)\ . \qquad (5)$$

It can be derived as sequence of $\pi/2$ rotations and 4-inversion

$$E_{ij}=\frac{1}{4}(1+\gamma_i^0\gamma_j^0)(1-\gamma_i^1\gamma_j^1)(1-\gamma_i^2\gamma_j^2)(1-\gamma_i^3\gamma_j^3)\gamma_i^{\text{FIVE}} \quad (6)$$

where

$$e^{\pm(\pi/4)\gamma_i^\mu\gamma_j^\mu}=\frac{1\pm\gamma_i^\mu\gamma_j^\mu}{\sqrt{2}}\ . \qquad (7)$$

The symbolic calculation was made by utilizing cloud version of SAGE. `www.sagemath.com` The resulting exchange matrix E_{ij} is not made from -1 only as expected, see the Table I, where I show only relevant channels. The rest of matrix is diagonal.

The Lagrangian Eq. (2) changes sign upon the subspace exchange E_{ij}. The exchange matrix E_{ij} is real, therefore the conjugated field $\psi^{N\dagger}$ obeys same exchange transformation as ψ^N, but E_{ij} anticommutes with γ^{ZERO}

$$\psi^{\dagger N\prime}(\ldots x_i\ldots x_j\ldots)=E_{ij}\psi^{\dagger N}(\ldots x_j\ldots x_i\ldots) \quad (8)$$

$$\Rightarrow\ \gamma^{\text{ZERO}}E_{ij}\gamma^{\text{ZERO}}=-E_{ij}\ . \qquad (9)$$

The issue will be resolved later by the time ordering post symmetry lowering.

The next task is to index the components of the $SO(N,3N)$ fields ψ_L^N,ψ_R^N according to the representations of the embedded group $SO(1,3)\oplus\cdots\oplus SO(1,3)$. We will prove that the exchange operator E has negative eigenvalue for exchange of the left fields only, as shown

$$
E=\left(\begin{array}{cccccccccccccccc}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&-1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{array}\right)\,,\qquad
E\left(\begin{array}{c}
\psi_{\dots\uparrow R\dots\uparrow R\dots}\\
\psi_{\dots\uparrow R\dots\downarrow L\dots}\\
\psi_{\dots\uparrow R\dots\uparrow L\dots}\\
\psi_{\dots\uparrow R\dots\downarrow R\dots}\\
\psi_{\dots\downarrow L\dots\uparrow R\dots}\\
\psi_{\dots\downarrow L\dots\downarrow L\dots}\\
\psi_{\dots\downarrow L\dots\uparrow L\dots}\\
\psi_{\dots\downarrow L\dots\downarrow R\dots}\\
\psi_{\dots\uparrow L\dots\uparrow R\dots}\\
\psi_{\dots\uparrow L\dots\downarrow L\dots}\\
\psi_{\dots\uparrow L\dots\uparrow L\dots}\\
\psi_{\dots\uparrow L\dots\downarrow R\dots}\\
\psi_{\dots\downarrow R\dots\uparrow R\dots}\\
\psi_{\dots\downarrow R\dots\downarrow L\dots}\\
\psi_{\dots\downarrow R\dots\uparrow L\dots}\\
\psi_{\dots\downarrow R\dots\downarrow R\dots}
\end{array}\right)=\left(\begin{array}{c}
\psi_{\dots\uparrow R\dots\uparrow R\dots}\\
\psi_{\dots\downarrow L\dots\uparrow R\dots}\\
\psi_{\dots\uparrow L\dots\uparrow R\dots}\\
\psi_{\dots\downarrow R\dots\uparrow R\dots}\\
\psi_{\dots\uparrow R\dots\downarrow L\dots}\\
-\psi_{\dots\downarrow L\dots\downarrow L\dots}\\
-\psi_{\dots\uparrow L\dots\downarrow L\dots}\\
\psi_{\dots\downarrow R\dots\downarrow L\dots}\\
\psi_{\dots\uparrow R\dots\uparrow L\dots}\\
-\psi_{\dots\downarrow L\dots\uparrow L\dots}\\
-\psi_{\dots\uparrow L\dots\uparrow L\dots}\\
\psi_{\dots\downarrow R\dots\uparrow L\dots}\\
\psi_{\dots\uparrow R\dots\downarrow R\dots}\\
\psi_{\dots\downarrow L\dots\downarrow R\dots}\\
\psi_{\dots\uparrow L\dots\downarrow R\dots}\\
\psi_{\dots\downarrow R\dots\downarrow R\dots}
\end{array}\right)
$$

TABLE I: The explicit form of the exchange operator in the form of matrix and its action on the wave function.

in the right hand side of the Table I. As we mentioned before, the spinor ψ^N has 2^{2N} components, we will mark them by N handedness indexes L or R and by N spin indexes $\uparrow$ or $\downarrow$.

The common definition of the left and right spinors is derived from their sign upon the $4N$ inversion; in spinor representation the γ^{FIVE} is the operator of the $4N$ inversion, since $\forall j\mu:\quad \gamma^{\text{FIVE}}\gamma^{\mu}_{j}\gamma^{\text{FIVE}}=-\gamma^{\mu}_{j}$. Then the components of the spinor Eq.(1) are selected by

$$
\psi^N_L=\frac{1-\gamma^{\text{FIVE}}_j}{2}\psi^N\ ,\quad \psi^N_R=\frac{1+\gamma^{\text{FIVE}}_j}{2}\psi^N\ , \qquad (10)
$$

and ψ^N_L will change the sign upon $4N$ inversion, while ψ^N_R will preserve the sign.

For every N we can extract the index L,R which belong to j-th subspace by making use of γ^{FIVE}_j - the operator of 4-inversion of the j-th subspace

$$
\psi_{\dots L\dots}\ =\ \frac{1-\gamma^{\text{FIVE}}_j}{2}\psi^N \qquad (11)
$$

$$
\psi_{\dots R\dots}\ =\ \frac{1+\gamma^{\text{FIVE}}_j}{2}\psi^N \qquad (12)
$$

for example the component of ψ branched to the product of only left fields is

$$
\psi_{L\dots L}=\left(\prod_{j=1}^{N}\frac{1-\gamma^{\text{FIVE}}_j}{2}\right)\psi^N\ . \qquad (13)
$$

The N spin indexes can be extracted according to the sign of the generator of rotation in $(j1,j2)$ plane:

$$
\psi_{\dots\uparrow\dots}\ =\ \frac{1+i\gamma^1_j\gamma^2_j}{2}\psi^N \qquad (14)
$$

$$
\psi_{\dots\downarrow\dots}\ =\ \frac{1-i\gamma^1_j\gamma^2_j}{2}\psi^N\ . \qquad (15)
$$

This way of labeling allows to write explicitly the action of the exchange operator Eq. (6) on the wave function, as shown in the right hand side of the Table I, and the same exactly equation holds for $\psi^{\dagger N}$.

At this time I don't have an alternative explanation to the sign change upon the exchange of the left fields. Let me only mention that the left and the right fields are not entirely symmetric upon group embedding (symmetry lowering). For example, it follows from the identity $\gamma^{\text{FIVE}}=\prod_j\gamma^{\text{FIVE}}_j$ that $\psi^N_{\text{Odd number of }L}$ belong to ψ^N_L and $\psi^N_{\text{Even number of }L}$ belong to ψ^N_R.

III. THE SPONTANEOUS SYMMETRY BREAKING FROM $SO(N,3N)$ TO $SO(1,3)$ BY THE SYMMETRIC TENSOR BOSON

Our next step is to find the field "Higgs boson" which can drive the spontaneous symmetry break from $SO(N,3N)$ to $SO(1,3)$. Let me formulate the following theorem: upon the spontaneous symmetry break the Higgs boson should branch into scalar of the low symmetry group.

It is enough to find the representations of the order parameter ("Higgs boson") in the high symmetry state $SO(N,3N)$ and in the low symmetry state $SO(1,3)$. There is no need to write explicitly the symmetry lowering formalism, this would be just re-writing text books.[16]

Let me mention that after I've worked out all the group theory analysis of this section, the similarity between the proposed below "Higgs boson" $h_{j\mu j'\mu'}$ and the space metric tensor became apparent. The symmetry lowering from $SO(N,3N)$ to $SO(1,3)$ breaks the $4N$-dimensional light cone to N orthogonal 4-dimensional light cones. It is still an open question if the gravitational waves can break the $SO(1,3)$ Lorenz symmetry. The broken Lorenz symmetry must have space metric possessing few light cones.[17, 18] Here we need $SO(N,3N)$ "gravitational" waves to break the symmetry to $SO(1,3)$, that is much easy than to break $SO(1,3)$ to the Finsler geometry.

The separation of one 4-dimensional subspace from $4N$-dimensional space

$$SO(N,3N) \to SO(N-1,3N-3) \oplus SO(1,3) \quad (16)$$

leads to following branching rules for spinors

$$\begin{aligned}[0,0,\ldots,1,0] &\to [0,\ldots,1,0][0,1]+[0,\ldots,0,1][1,0]\\ [0,0,\ldots,0,1] &\to [0,\ldots,1,0][1,0]+[0,\ldots,0,1][0,1]\end{aligned}$$

and for the traceless symmetric tensor

$$\begin{aligned}[2,0,\ldots,0,0] &\to [0,\ldots,0,0][0,0]+[1,0,\ldots,0][1,1]\\ &+ [2,\ldots,0,0][0,0]+[0,\ldots,0,0][2,2] .\end{aligned}$$

The last equation means that $SO(N,3N)$ traceless symmetric tensor branches to scalar, product of vectors, $SO(N-1,3N-3)$ traceless symmetric tensor and SO(1,3) traceless symmetric tensor. It is similar to the theorem stating that adjoint representation branches to sum of adjoint and product of vectors.[15] In this way we introduce the Higgs field

$$h_{j\mu j'\mu'} = [2,0,\ldots,0] , \quad h_{j\mu j'\mu} = h_{j'\mu' j\mu} , \quad \mathrm{Tr} h = 0 \quad (17)$$

For the N-th subspace to break off the $SO(N,3N)$ group, the sum of four diagonal elements should acquire the observation value

$$\sum_\mu h_{N\mu N\mu} = \text{const}; \quad (18)$$

it becomes scalar under the symmetry lowering. The remaining components of the tensor are $h_{N\mu N\mu'}$ and they make traceless symmetric tensor $[2,2]$.

For the $SO(N,3N) \to \sum SO(1,3) \sum \cdots \sum SO(1,3)$ symmetry lowering (it can be also called as the group embedding) the Higgs field should acquire $N-1$ observation values

$$\forall j \quad \sum_\mu h_{j\mu j\mu} = v_j = \text{const} \quad (19)$$

constrained by $\mathrm{Tr} h = 0$.

The coupling between the Higgs field h and the matter field ψ before the symmetry lowering is

$$\mathcal{L}_{\text{int}} = (\psi^{\dagger N} \gamma^{\text{ZERO}} \gamma^\mu_j \psi^N) h_{j\mu j'\mu'} (\psi^{\dagger N} \gamma^{\text{ZERO}} \gamma^{\mu'}_{j'} \psi^N) . \quad (20)$$

The symmetry lowering of the field h will eventually drive the symmetry lowering of the matter field ψ, however the explicit calculation of the ground state post symmetry lowering is beyond the scope of this work.

The general formula for a bi-spinor branching upon the group embedding can be straightforwardly derived by induction; it goes according to chirality labels in the ψ^N index. We need explicit formulas for $\psi^N, \psi^{\dagger N}$ branching upon the symmetry lowering:

$$\psi^N_{\alpha_1\ldots\alpha_N}(x) \to T \prod_j \psi_{j\alpha_j}(x_j) \quad (21a)$$

$$\psi^{\dagger N}_{\alpha_1\ldots\alpha_N}(x) \to T \prod_j \psi^\dagger_{j\alpha_j}(x_j) \quad (21b)$$

where $\psi^\dagger_{j\alpha}, \psi_{j\alpha}$ are $SO(1,3)$ operators. The time ordering operator T is required to remove ambiguity related to commutation rules between $\psi_{j\alpha}$ and $\psi_{j'\alpha'}$.

The $SO(N,3N)$ exchange symmetry is preserved post symmetry lowering by declaring all the left hand fields to be anticommuting and all the right hand fields to be commuting variables independently of spin.

$$\{\psi_L, \psi_L\} = \{\psi^\dagger_L, \psi^\dagger_L\} = 0 , \quad (22a)$$

$$[\psi_R, \psi_R] = [\psi^\dagger_R, \psi^\dagger_R] = 0 , \quad (22b)$$

$$[\psi_R, \psi_L] = [\psi^\dagger_R, \psi^\dagger_L] = 0 . \quad (22c)$$

These equations are the analog of Eq. (5) post symmetry lowering.

The derivation of the Lagrangian $\mathcal{L}_N$ in the lower symmetry state requires accurate counting of powers of γ^{FIVE} coming from the decomposition of both γ^{ZERO} and γ^μ_j:

$$\begin{aligned}\mathcal{L}_N &= T \sum_{j\mu} \prod_{k=1}^{j-1} \psi^\dagger_k \gamma^0 (\gamma^{\text{FIVE}})^{k-1} \psi_k \\ &\times \psi^\dagger_j \gamma^0 (\gamma^{\text{FIVE}})^{j-1} \gamma^\mu i\partial_\mu \psi_j \prod_{k=j+1}^{N} \psi^\dagger_k \gamma^0 (\gamma^{\text{FIVE}})^k \psi_k ,\end{aligned}$$

where γ-matrices and ψ-operators belong to $SO(1,3)$, and $\psi^\dagger_k$ creates particle in k-th instance of $SO(1,3)$ space. Taking ψ and $\psi^\dagger$ as independent variables we can change the sign of ψ_{jL} for even j and arrive at

$$\begin{aligned}\mathcal{L}_N &= T \sum_{j\mu} \prod_{k=1}^{j-1} \psi^\dagger_k \gamma^0 \psi_k \\ &\times \psi^\dagger_j \gamma^0 \gamma^\mu i\partial_\mu \psi_j \prod_{k=j+1}^{N} \psi^\dagger_k \gamma^0 \gamma^{\text{FIVE}} \psi_k , \end{aligned} \quad (23)$$

that is the final expression for the Lagrangian.

Now we are ready to integrate out the fields which belong to the split-out subspaces $j,k > 1$. We will preserve only one instance of the $SO(1,3)$ world, $j = k = 1$, and calculate the effective action for the lower symmetry state. The effective Lagrangian is calculated by tracing the partition function over $\psi_2 \ldots \psi_N$ degrees of freedom

$$e^{i\int d^4x \mathcal{L}} = \langle | e^{i\mathcal{S}_N} | \rangle . \quad (24)$$

The dimensionality of the remaining field $[\psi]$ should be $[x]^{-\frac{3}{2}}$ as in regular QED, therefore fields $[\psi_j]$ with $j > 1$ must have the dimensionality of $[x]^{-2}$. The commutation rules for the fields in the $j = 1$ space are

$$\{\psi_{sL}(\vec{r},t), \psi^\dagger_{s'L}(\vec{r}',t)\} = \delta_{ss'} \delta^3(\vec{r}-\vec{r}') \quad (25a)$$

$$[\psi_{sR}(\vec{r},t), \psi^\dagger_{s'R}(\vec{r}',t)] = \delta_{ss'} \delta^3(\vec{r}-\vec{r}') . \quad (25b)$$

and for $j > 1$

$$\{\psi_{jsL}(x), \psi^\dagger_{js'L}(x')\} = \delta_{ss'} \delta^4(x-x') \quad (25c)$$

$$[\psi_{jsR}(x), \psi^\dagger_{js'R}(x')] = \delta_{ss'} \delta^4(x-x') . \quad (25d)$$

This guarantee convergence of the $\int d^4x \psi_j^\dagger \psi_j$.

Terms proportional to $\psi^\dagger\gamma^0\psi$ require special attention because they mix commuting and anticommuting fields:

$$\psi^\dagger\gamma^0\psi = \psi_R^\dagger\gamma^0\psi_L + \psi_L^\dagger\gamma^0\psi_R \ .$$

It can be proved by the method of canonical quantization, that the supercharge Q comes out from following

$$\forall k > 1 \quad \int d^4x \, \langle | \, \psi_k^\dagger\gamma^0\psi_k \, | \rangle = Q \tag{26a}$$

The operator Q is swaps the commutation properties of the field operators, besides $Q^2 = 1$. In the present theory it must be a scalar, as opposite to the SUSY[12, 13], where it is defined as a spinor.

Remaining integrals in Eq. (23) can be replaced by the ground state expectation values

$$\forall k > 1 \quad \int d^4x \, \langle | \, \psi_k^\dagger\gamma^0\gamma^{\text{FIVE}}\psi_k \, | \rangle = QW_k \ , \tag{26b}$$

$$\int d^4x \, \langle | \, \psi_k^\dagger\gamma^0\gamma^\mu i\partial_\mu\psi_k \, | \rangle = V_k \ , \tag{26c}$$

and the effective $SO(1,3)$ Lagrangian becomes

$$\mathcal{L} = \psi^\dagger\gamma^0\gamma^\mu i\partial_\mu\psi - \Lambda\psi^\dagger\gamma^0 Q\psi \tag{27}$$

where the index $j = k = 1$ of the ψ-operator is omitted. It contains explicit boson-fermion swap operator Q. The parameter Λ accumulates all terms with $j > 1$ in Eq.(23) in the ground state:

$$-\Lambda = \sum_{j>1} \prod_{k=2}^{j-1} W_k V_j \xrightarrow{N\to\infty} \frac{V}{1-W} \ , \tag{28}$$

where W_k and V_k are approximated by constants. This ground state will depend on the coupling between the symmetry breaking field h and the material field ψ, this calculation is beyond the scope of this work. The standard model predicts that the electron mass comes from the symmetry lowering of the unified electro-weak Lagrangian[16]. The Eq. (28) gives the alternative way to calculate the spinor mass, because it converges to the N-independent constant.

IV. CONCLUSION REMARKS AND NEXT STEPS

The left spinor at high energy is associated with an electron and the right spinor with a positron[16]§VII.2, therefore we come up with the Lorentz-invariant theory describing commuting positrons and anti-commuting electrons. This simple picture does not work for all energies. Two additional papers will take forward the obtained result. The massive supersymmetric Lagrangian with the statistics flip operator is introduced and quantized in the second paper.[11] It arrives at the Lorentz-invariant and $\hat{C}\hat{P}\hat{T}$-invariant QED with commuting particles and anticommuting anti-particles. It provides necessary modifications in the diagram technique to accommodate the super-symmetry. Clearly, all the single particle processes (e.g. electron-positron annihilation) stay the same, as of today QED.

The dipositronium Zeeman effect is calculated in the third paper.[19] It identifies the fermionic antiparticles with electrons and the bosonic particles with positrons; then it predicts the linear Zeeman split of the dipositronium ground state.

The world made from bosonic charges becomes unstable.[20] The 4-inversion takes our stable fermionic world to the unstable bosonic world; the 4-mirror image of our universe cannot survive. In other words this work predicts instability of the antimatter.

This paper does not check the compatibility of the proposed theory with the standard model. The starting point of present theory is the assumption that initially world was one particle of the very large dimension; as large as there are particles in the world today. Once the symmetry is lowered to $SO(1,3)$ we arrive to the field operators capable to create as many particles as need. It is possible that some of degrees of freedom of the original field went into internal degrees of freedom of the standard model.

At the end, the supersymmetric Lagrangian with the scalar supercharge satisfying the Coleman-Mandula theorem and bypassing the Pauli principle is not proposed *ad hoc*. It was derived by the symmetry lowering from $SO(N,3N)$ to $SO(1,3)$, where N is the number of particles.

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