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Nonexistence of the periodic table for antimatter

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Assume the exchange of indistinguishable half-integer spin particles and the CPT transformation (4-inversion) do not commute. This explains the matter-antimatter (baryon) asymmetry of the universe (BAU), because antimatter would be unstable. Extension of the periodic table to the antimatter sector will not be possible for the same reason.

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I. INTRODUCTION TO STABILITY OF MATTER

The periodic table of elements is the most fundamental evidence for the quantum mechanics and for the exclusion principle. There is no way to explain the periodic filling of atomic orbitals without the exclusion principle. The Coulomb repulsion is not enough for holding the stability of matter[1, 2].

There are quite a few theoretical quests and experimental efforts to fill the periodic table for antimatter, however today it does not go beyond the antihydrogen atom.[3, 4] Partial explanation is related of course to experimental difficulties to produce enough density of the antihelium nucleus in order to form antihelium atoms. [5–7] What if the exclusion principle does not hold for antimatter?

In such a theory the CPT invariance (which says that matter is identical to antimatter) will work for single particle and for any number of distinguishable particles. For example a hydrogen atom and an antihydrogen atom should be identical in all observed properties[8]. However the CPT invariance will not work for a set of two or more indistinguishable particles. An antihydrogen molecule and an antihelium atom would be very different from their matter counterparts.

In this theory there is antihydrogen, but there is no periodic table for antielements. Few light antiatoms will have bound state with positron condensate in lowest S-wave state. Heavy antiatoms will be unstable and will not be able to form solid antimatter. Antimatter will not be able to form any cosmological object balanced against gravitational collapse by degeneracy pressure.

Relatively small amount of observed antimmatter in cosmic rays can be evidence for the certain scenario of the baryogenesis[9, 10], the CPT invariance violation[11], models where antimatter cannot be observed because it is stored in black holes. The lack of exclusion principle for antimatter is the alternative solution to the paradox of matter antimatter asymmetry. It does not require any of Sakharov's conditions.[12]

It therefore highly important to collect experimental data on antimatter statistics. For example, the interaction of antiparticles was recently observed in the beam of antiprotons from collisions of heavy elements.[13, 14] The correlation of antiprotons was found similar to that of protons indicating same statistics for antiprotons and protons. However it is also similar to correlation of S=1 deuterons because Coulomb interaction clouds the exchange.[15]

The statistics of positrons can be directly probed by positron-positron Moller scattering. None was able to measure it so far because of low luminosity of positron beams from beta-decay sources.[4] Zeeman effect in the dipositronium molecule[16] also depends on sign of positron-positron exchange, but the experimental difficulty comes from the short life time of dipositronium in magnetic field.[4]

We see lack of experimental evidences proving anti-commutation of positrons, and will discuss the antiworld made from commuting positrons. The ground state energy of Z positive and Z negative commuting charged particles is estimated[17, 18] as $E \sim -Z^{7/5}$. It satisfies the stability condition of the first kind $E > -\infty$, meaning that these particles can form a bound state. However it fails to satisfy the stability condition of the second kind $E > -CZ$, where $C > 0$ is some constant. The stability condition of the second is necessarily (not sufficient) for existence of the thermodynamic limit.[2]

The positronic structure of antiatom is yet another example of the ground state calculation with commuting charges. The calculation is done in Sec. VI for antinuclear of charge $-|e|Z$ surrounded by Z positrons in S-state. The model predicts $\rho_{AA}(Z) \sim Z^4$ while $\rho_{TF}(Z) \sim Z^2$ where $\rho_{AA}(Z)$ is density of positrons near the origin of an antiatom and $\rho_{TF}(Z)$ is the density of electrons in Thomas-Fermi model of an atom.

The stability of atoms is limited (i) by nuclear decay, (ii) by stability of vacuum in presence of high electric field, (iii) by interaction between nuclear and shell (e.g. electron capture) and (iv) by termination of electron spectrum at critical charge.[19–21] Scenarios (i), (ii), (iv) depend weakly on statistics of charges in the atomic shell, and should give approximately same maximal $Z_{\max}$ of antiatoms, as of atoms. In the case (iii) an antiatom should be stable up to critical density of positrons equal to density of electrons in a largest stable atom. Stable atoms are observed up to $Z \sim 100$ and therefore stable antiatoms should exist up to $Z_{\max} \sim 10$.

The plan of the paper is as follows. Calculation of ground state of an antiatom in Sec. VI is preceded by consistency check for the assumption that matter and antimatter belong to opposite symmetry classes relatively to particle permutations. We prove in Sec. II that energy of the system is positively defined for any number of particles and antiparticles, and remains positively defined under the CPT transformation. Charge and Green function are proven to be odd relatively to the CPT transformation. We prove CPT invariance of ordered products of quantum operators in Sec. III.

Interaction of indistinguishable particles is not CPT invariant in present theory; in other words positron is the CPT image of electron, but interaction of two positrons is not the CPT image of interaction of two electrons. The analysis in Sec. IV proves that Wick theorem is not CPT invariant. Straightforwardly interaction diagrams are not CPT invariant too. We summarize all theoretical postulates in Sec. V, and the entire paper in Sec. VII.

II. CPT INVARIANCE OF ENERGY, CHARGE AND GREEN FUNCTION.

Let's review the quantum theory where the CPT transformation (4-inversion, denoted as $\hat{I}_4$) converts half integer spin particles to antiparticles and vice versa

$$\hat{I}_4 b = f^\dagger \ , \quad \hat{I}_4 b^\dagger = f \ , \quad \hat{I}_4 f = b^\dagger \ , \quad \hat{I}_4 f^\dagger = b \ . \tag{1}$$

Here $b_i^\dagger |0\rangle$ and $f_i^\dagger |0\rangle$ create a particle and an antiparticle in i-th state. In the proposed theory the CPT transformation does not commute with the particle exchange, for example: $b_j b_i = b_i b_j$ but $\hat{I}_4(b_j b_i) \neq \hat{I}_4(b_i b_j)$. Specifically let's focus on the situation where

$$\begin{aligned} \hat{I}_4(b_i b_j b_k \dots b_l^\dagger b_m^\dagger b_n^\dagger) &= f_n f_m f_l \dots f_k^\dagger f_j^\dagger f_i^\dagger \ , \\ \hat{I}_4(b_i b_j b_k \dots b_l^\dagger b_n^\dagger b_m^\dagger) &= -f_m f_n f_l \dots f_k^\dagger f_j^\dagger f_i^\dagger \ . \end{aligned} \tag{2}$$

States $b_m^\dagger b_n^\dagger |0\rangle$ and $b_n^\dagger b_m^\dagger |0\rangle$ are identical unless someone try to invert the 4-dimensional space-time.

The exchange of particles between states i, j is given by the operator

$$\hat{E}_{ji} b_j b_i = b_i b_j \ , \quad \hat{E}_{ji} f_j f_i = f_i f_j \ . \tag{3}$$

The basic proposal of the theory is the anticommutation between the exchange and the CPT transformation

$$\hat{E}_{ji} \hat{I}_4 = -\hat{I}_4 \hat{E}_{ji} \ . \tag{4}$$

This is obvious violation of Pauli exclusion principle, because it predicts the ensemble of like particles to be mixture of symmetric and antisymmetric classes.[22] It also means that positrons cannot be regarded as holes in electron sea.[23]

Let's check that the energy remains positive and charge remains odd for a system of spin 1/2 fields with above properties. The textbook expressions[24, §3] for the energy and the charge are (omit spin indexes)

$$\mathcal{E} = \sum_{\vec{p}} \varepsilon_{\vec{p}} (b_{\vec{p}}^\dagger b_{\vec{p}} - f_{\vec{p}} f_{\vec{p}}^\dagger) = \sum_{\vec{p}} \varepsilon_{\vec{p}} (b_{\vec{p}}^\dagger b_{\vec{p}} + f_{\vec{p}}^\dagger f_{\vec{p}}) - C \tag{5}$$

$$\mathcal{Q} = \sum_{\vec{p}} (b_{\vec{p}}^\dagger b_{\vec{p}} + f_{\vec{p}} f_{\vec{p}}^\dagger) = \sum_{\vec{p}} (b_{\vec{p}}^\dagger b_{\vec{p}} - f_{\vec{p}}^\dagger f_{\vec{p}}) + C' \ . \tag{6}$$

These equations are valid when the antiparticle field $f_{\vec{p}}$ is anticommuting. No conclusion can be derived about statistics of the particle field $b_{\vec{p}}$ from Eqs. (5,6). Therefore the next step is to verify the CPT invariance of energy and charge for commuting particle field $b_{\vec{p}}$.

Looking ahead say that in present theory particles with the "positive" frequency must be commuting, while antiparticles with "negative" frequency must be anticommuting. The modern physics describes atoms in our universe as made from anticommuting particles with "positive" frequency. In present theory our universe is predominantly made from anticommuting particles with "negative" frequency.

Upon the CPT transformation we expect the energy and charge operators to be preserved and commutation rules to be kept valid. The charge should be always # particles - # antiparticles; then the charge will change sign upon exchange of particle and antiparticles. Besides, $\hat{I}_4 \mathcal{E}$ flips the sign of $\varepsilon_{\vec{p}}$ in Eq. (5) because it counts for time reversion in spinor Hamiltonian $i\partial/\partial t$. In order to satisfy

$$\hat{I}_4 \mathcal{E} = \mathcal{E} \ , \quad \hat{I}_4 \mathcal{Q} = \mathcal{Q} \ . \tag{7}$$

we get additional rules

$$\hat{I}_4(b_j b_i^\dagger) = f_i f_j^\dagger \ , \quad \hat{I}_4(b_i^\dagger b_j) = -f_j^\dagger f_i \tag{8}$$

and vise versa. Combining them together

$$\left. \begin{array}{l} \hat{I}_4([b_j, b_i^\dagger]) = \{f_i, f_j^\dagger\} \\ \hat{I}_4(\delta_{ij}) = \delta_{ij} \\ [b_i, b_j^\dagger] = \delta_{ij} \end{array} \right\} \Rightarrow \ \{f_i, f_j^\dagger\} = \delta_{ij} \ . \tag{9}$$

we prove the CPT invariance of commutation rules.

The comparison of Eqs. (8,9) with the textbook case of anticommuting particle fields is summarized in Table I. First column labels variants of the theory; 2nd column gives the commutation relations between particles and between antiparticles; 3rd column is short form of Eqs. (5,6) for half integer spin fields (#1)-(#4) and integer spin fields (#5),(#6). Next (4th) column describes the change of the sign in Eqs. (5,6) due to change of $\partial/\partial t$ upon the time reversion. The 5th column shows transformation of energy and charge operators upon the CPT transformation $\hat{I}_4$. The last 6th column shows whether $\hat{I}_4$ and $\hat{E}$ commute or anticommute.

There are two approaches to explain the CPT invariance for anticommuting particles and anticommuting antiparticles, see (#1) and (#2). In former case (#1) one

Var	Commutators	CPT-transformation		Eqs. for $\mathcal{E}$ and $\mathcal{Q}$	$\hat{I}_4\partial/\partial t$	Energy&Charge	Exchange
#1	$\{b_{\vec p}, b_{\vec q}^\dagger\} = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec p}b_{\vec q}^\dagger) = f_{\vec p}^\dagger f_{\vec q}$	$\hat{I}_4(f_{\vec p}^\dagger f_{\vec q}) = b_{\vec p}b_{\vec q}^\dagger$	$\mathcal{E} = \sum \varepsilon_{\vec p}(b^\dagger b - ff^\dagger)$	$\varepsilon_{\vec p} \to -\varepsilon_{\vec p}$	$\hat{I}_4\mathcal{E} = \mathcal{E}$	$[\hat{I}_4, \hat{E}] = 0$
	$\{f_{\vec q}, f_{\vec p}^\dagger\} = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec q}^\dagger b_{\vec p}) = f_{\vec q}f_{\vec p}^\dagger$	$\hat{I}_4(f_{\vec q}f_{\vec p}^\dagger) = b_{\vec q}^\dagger b_{\vec p}$	$\mathcal{Q} = \sum q_0(b^\dagger b + ff^\dagger)$	$q_0 \to q_0$	$\hat{I}_4\mathcal{Q} = \mathcal{Q}$	
#2	$\{b_{\vec p}, b_{\vec q}^\dagger\} = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec p}b_{\vec q}^\dagger) = f_{\vec q}f_{\vec p}^\dagger$	$\hat{I}_4(f_{\vec q}f_{\vec p}^\dagger) = b_{\vec p}b_{\vec q}^\dagger$	$\mathcal{E} = \sum \varepsilon_{\vec p}(b^\dagger b - ff^\dagger)$	$\varepsilon_{\vec p} \to -\varepsilon_{\vec p}$	$\hat{I}_4\mathcal{E} = -\mathcal{E}$	$[\hat{I}_4, \hat{E}] = 0$
	$\{f_{\vec q}, f_{\vec p}^\dagger\} = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec q}^\dagger b_{\vec p}) = f_{\vec p}^\dagger f_{\vec q}$	$\hat{I}_4(f_{\vec p}^\dagger f_{\vec q}) = b_{\vec q}^\dagger b_{\vec p}$	$\mathcal{Q} = \sum q_0(b^\dagger b + ff^\dagger)$	$q_0 \to q_0$	$\hat{I}_4\mathcal{Q} = -\mathcal{Q}$	
#3	$[b_{\vec p}, b_{\vec q}^\dagger] = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec p}b_{\vec q}^\dagger) = f_{\vec p}^\dagger f_{\vec q}$	$\hat{I}_4(f_{\vec p}^\dagger f_{\vec q}) = b_{\vec p}b_{\vec q}^\dagger$	$\mathcal{E} = \sum \varepsilon_{\vec p}(b^\dagger b - ff^\dagger)$	$\varepsilon_{\vec p} \to -\varepsilon_{\vec p}$	$\hat{I}_4\mathcal{E} = -\mathcal{E}$	$\{\hat{I}_4, \hat{E}\} = 0$
	$\{f_{\vec q}, f_{\vec p}^\dagger\} = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec q}^\dagger b_{\vec p}) = -f_{\vec q}f_{\vec p}^\dagger$	$\hat{I}_4(f_{\vec q}f_{\vec p}^\dagger) = -b_{\vec q}^\dagger b_{\vec p}$	$\mathcal{Q} = \sum q_0(b^\dagger b + ff^\dagger)$	$q_0 \to q_0$	$\hat{I}_4\mathcal{Q} = -\mathcal{Q}$	
#4	$[b_{\vec p}, b_{\vec q}^\dagger] = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec p}b_{\vec q}^\dagger) = f_{\vec q}f_{\vec p}^\dagger$	$\hat{I}_4(f_{\vec q}f_{\vec p}^\dagger) = b_{\vec p}b_{\vec q}^\dagger$	$\mathcal{E} = \sum \varepsilon_{\vec p}(b^\dagger b - ff^\dagger)$	$\varepsilon_{\vec p} \to -\varepsilon_{\vec p}$	$\hat{I}_4\mathcal{E} = \mathcal{E}$	$\{\hat{I}_4, \hat{E}\} = 0$
	$\{f_{\vec q}, f_{\vec p}^\dagger\} = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec q}^\dagger b_{\vec p}) = -f_{\vec p}^\dagger f_{\vec q}$	$\hat{I}_4(f_{\vec p}^\dagger f_{\vec q}) = -b_{\vec q}^\dagger b_{\vec p}$	$\mathcal{Q} = \sum q_0(b^\dagger b + ff^\dagger)$	$q_0 \to q_0$	$\hat{I}_4\mathcal{Q} = \mathcal{Q}$	
#5	$[b_{\vec p}, b_{\vec q}^\dagger] = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec p}b_{\vec q}^\dagger) = -f_{\vec p}^\dagger f_{\vec q}$	$\hat{I}_4(f_{\vec p}^\dagger f_{\vec q}) = -b_{\vec p}b_{\vec q}^\dagger$	$\mathcal{E} = \sum \varepsilon_{\vec p}(b^\dagger b + ff^\dagger)$	$\varepsilon_{\vec p} \to \varepsilon_{\vec p}$	$\hat{I}_4\mathcal{E} = -\mathcal{E}$	$[\hat{I}_4, \hat{E}] = 0$
	$[f_{\vec q}, f_{\vec p}^\dagger] = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec q}^\dagger b_{\vec p}) = -f_{\vec q}f_{\vec p}^\dagger$	$\hat{I}_4(f_{\vec q}f_{\vec p}^\dagger) = -b_{\vec q}^\dagger b_{\vec p}$	$\mathcal{Q} = \sum q_0(b^\dagger b - ff^\dagger)$	$q_0 \to -q_0$	$\hat{I}_4\mathcal{Q} = -\mathcal{Q}$	
#6	$[b_{\vec p}, b_{\vec q}^\dagger] = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec p}b_{\vec q}^\dagger) = f_{\vec q}f_{\vec p}^\dagger$	$\hat{I}_4(f_{\vec p}^\dagger f_{\vec q}) = b_{\vec q}^\dagger b_{\vec p}$	$\mathcal{E} = \sum \varepsilon_{\vec p}(b^\dagger b + ff^\dagger)$	$\varepsilon_{\vec p} \to \varepsilon_{\vec p}$	$\hat{I}_4\mathcal{E} = \mathcal{E}$	$[\hat{I}_4, \hat{E}] = 0$
	$[f_{\vec q}, f_{\vec p}^\dagger] = \delta_{\vec p\vec q}$	$\hat{I}_4(b_{\vec q}^\dagger b_{\vec p}) = f_{\vec p}^\dagger f_{\vec q}$	$\hat{I}_4(f_{\vec q}f_{\vec p}^\dagger) = b_{\vec p}b_{\vec q}^\dagger$	$\mathcal{Q} = \sum q_0(b^\dagger b - ff^\dagger)$	$q_0 \to -q_0$	$\hat{I}_4\mathcal{Q} = \mathcal{Q}$	

TABLE I: Variants of consistent definition of commutation rules and the CPT transformation: valid commutation rules are transformed to other valid commutation rules. (#1) and (#2) are classical text book variants for half-integer spin fields. (#3) and (#4) are rules for supersymmetric fields. (#5) and (#6) are possible rules for commuting integer spin fields. (## 1,3,5) keep order of operators, (## 2,4,6) change order of operators. (##2,3,5) change the sign of energy and charge and should be discarded. (#4) is brought in this paper as chosen by nature together with (#6).

keeps the order of operators in the CPT transformation; the other approach (#2) is to change the order of operators in the CPT transformation, but this leads to unphysical change in sign of energy and charge. There are also two approaches to demonstrate the CPT invariance for commuting particles and anticommuting antiparticles, see (#3) and (#4) in Table I. Then Eqs. (8,9) follow approach (#4); it flips the sign of $\varepsilon_{\vec p}$ and change the order of operators, see also Eq. (2). We will see in a moment that only methods (#2) and (#4) give correct transformation of the Green function. This take me to speculative conclusion that only method (#4) of present work is correct and should be chosen by nature.

The commuting integer spin fields also have two variants of the CPT transformation preserving commutation rules, see (#5) and (#6) in Table I. Textbook expressions for energy and charge in (#5) and (#6) for integer spin fields[24, §3] differs from Eqs. (5,6) by the sign and transformation rules. Only the method (#6) keeps energy and charge and it is chosen by nature.

As pointed out in Landau course[19, §26], "a particle with spin s may be regarded as composed of $2s$ particles with spin $\frac{1}{2}$". Therefore the action of $f^\dagger b^\dagger$ from (#4) creates integer spin anticommuting field violating (#6). We conclude therefore that quantized vector and tensor fields cannot be regarded as composite, and should always have dedicated creation and annihilation operators.

The invariance of the Green function is to be proven for one-body case, see Table II. The CPT transformation of the Green function in the case #4 is obtained

$$\hat{I}_4 G(t, t') = -G(t, t') \tag{10}$$

that is in agreement with generic $G(-t, -t') = -G(t, t')$ inside the light cone as noted by Pauli[25]. The invariance of the scattering matrix follows, thus bypassing the geometrical arguments in favor of Pauli exclusion principle[26].

The most subtle thing is the CPT transformation of the powers of particle operators b_j and $b_j^\dagger$ because of the exclusion principle for antiparticle operators $f_j^\dagger$ and f_j. It will not allow any power of $f_j^\dagger$ and f_j. We therefore should admit the rule

$$\hat{I}_4(b_j^2) = \hat{I}_4(b_j^{\dagger 2}) = 0\ . \tag{11}$$

For the 1st glance this makes the theory inconsistent, but actually the CPT transformation does not exist in real world. So we should not be concerned with the energy and charge loss by the above rule. Alternatively we can say that the CPT invariance is broken in two ways: i) it does not commute with the exchange, ii) it converts multiple occupancy to zero occupancy.

III. INVARIANCE OF ORDERED PRODUCTS

The ordered product of quantum fields here means that product has normal order (all creation operators are to the left of all annihilation operators) or antinormal order (all annihilation operators are placed to the left of creation operators). The exchange of particles (by action of the operator $\hat{E}_{ij}$) means by the definition that the order is preserved.

Regarding exchange in a product of many field operators, let's first take example of $\hat{E}_{ij} b_i b_k b_j = b_j b_k b_i$:

$$\hat{E}_{ij} = \hat{E}_{ik}\hat{E}_{ij}\hat{E}_{kj}, \quad \hat{E}_{ik}\hat{E}_{ij}\hat{E}_{kj}\hat{I}_4 = -\hat{I}_4\hat{E}_{ik}\hat{E}_{ij}\hat{E}_{kj} \tag{12}$$

We conclude that $\hat{I}_4$ commutes with a product of even number of exchange operators E_{ij} and anticommutes with a product of odd number of exchange operators. For two states $|1\rangle$ and $|2\rangle$, which are i) made from vacuum by an ordered product of field operators, ii) not having

##	$-iT\psi(x)\psi^\dagger(x') = -i\psi(x)\psi^\dagger(x')\big\|_{t>t'} + i\psi^\dagger(x')\psi(x)\big\|_{t<t'}$	$-iT\langle 0\| \psi(x)\psi^\dagger(x') \|0\rangle$	$\hat{I}_4 G(t,t')$
orig	$\left[e^{-i(t-t')\varepsilon_{\vec{p}}} b_{\vec{p}} b^\dagger_{\vec{p}} + e^{i(t-t')\varepsilon_{\vec{p}}} f^\dagger_{\vec{p}} f_{\vec{p}}\right]_{t>t'} - \left[e^{-i(t-t')\varepsilon_{\vec{p}}} b^\dagger_{\vec{p}} b_{\vec{p}} + e^{i(t-t')\varepsilon_{\vec{p}}} f_{\vec{p}} f^\dagger_{\vec{p}}\right]_{t<t'}$	$e^{-i(t-t')\varepsilon_{\vec{p}}}\big\|_{t>t'} - e^{i(t-t')\varepsilon_{\vec{p}}}\big\|_{t<t'}$	-
#1	$\left[e^{i(t-t')\varepsilon_{\vec{p}}} f^\dagger_{\vec{p}} f_{\vec{p}} + e^{-i(t-t')\varepsilon_{\vec{p}}} b_{\vec{p}} b^\dagger_{\vec{p}}\right]_{t<t'} - \left[e^{i(t-t')\varepsilon_{\vec{p}}} f_{\vec{p}} f^\dagger_{\vec{p}} + e^{-i(t-t')\varepsilon_{\vec{p}}} b^\dagger_{\vec{p}} b_{\vec{p}}\right]_{t>t'}$	$-e^{i(t-t')\varepsilon_{\vec{p}}}\big\|_{t>t'} + e^{-i(t-t')\varepsilon_{\vec{p}}}\big\|_{t<t'}$	$-G^*(t,t')$
#2	$\left[e^{i(t-t')\varepsilon_{\vec{p}}} f_{\vec{p}} f^\dagger_{\vec{p}} + e^{-i(t-t')\varepsilon_{\vec{p}}} b^\dagger_{\vec{p}} b_{\vec{p}}\right]_{t<t'} - \left[e^{i(t-t')\varepsilon_{\vec{p}}} f^\dagger_{\vec{p}} f_{\vec{p}} + e^{-i(t-t')\varepsilon_{\vec{p}}} b_{\vec{p}} b^\dagger_{\vec{p}}\right]_{t>t'}$	$-e^{-i(t-t')\varepsilon_{\vec{p}}}\big\|_{t>t'} + e^{i(t-t')\varepsilon_{\vec{p}}}\big\|_{t<t'}$	$-G(t,t')$
#3	$\left[e^{i(t-t')\varepsilon_{\vec{p}}} f^\dagger_{\vec{p}} f_{\vec{p}} + e^{-i(t-t')\varepsilon_{\vec{p}}} b_{\vec{p}} b^\dagger_{\vec{p}}\right]_{t<t'} + \left[e^{i(t-t')\varepsilon_{\vec{p}}} f_{\vec{p}} f^\dagger_{\vec{p}} + e^{-i(t-t')\varepsilon_{\vec{p}}} b^\dagger_{\vec{p}} b_{\vec{p}}\right]_{t>t'}$	$e^{i(t-t')\varepsilon_{\vec{p}}}\big\|_{t>t'} + e^{-i(t-t')\varepsilon_{\vec{p}}}\big\|_{t<t'}$	-
#4	$\left[e^{i(t-t')\varepsilon_{\vec{p}}} f_{\vec{p}} f^\dagger_{\vec{p}} - e^{-i(t-t')\varepsilon_{\vec{p}}} b^\dagger_{\vec{p}} b_{\vec{p}}\right]_{t<t'} + \left[e^{i(t-t')\varepsilon_{\vec{p}}} f^\dagger_{\vec{p}} f_{\vec{p}} - e^{-i(t-t')\varepsilon_{\vec{p}}} b_{\vec{p}} b^\dagger_{\vec{p}}\right]_{t>t'}$	$-e^{-i(t-t')\varepsilon_{\vec{p}}}\big\|_{t>t'} + e^{i(t-t')\varepsilon_{\vec{p}}}\big\|_{t<t'}$	$-G(t,t')$

TABLE II: Four variants of the CPT transformation of time ordered product and Green function. Variants of theory are defined in the same way as in Table I. The line labeled "orig" shows definition of Green function for half integer spin fields. Spin and spacial dependency are taken as common in all cases and are not shown.

powers of b_j and $b_j^\dagger$ above one, one has

$$|2\rangle = \hat{E}^n |1\rangle \quad \Rightarrow \quad \hat{I}_4 |2\rangle = (-1)^n \hat{I}_4 |1\rangle \ . \tag{13}$$

It means that the relative sign of ordered products post CPT transformation is changed if and only if these states are related by the odd number of exchanges.

There is no way to define the sign for the CPT transformation of a single state $|1\rangle$. Once this sign is set, the CPT transformations of all states $|2\rangle$ related to $|1\rangle$ by Eq. (13) will have unambiguous sign. Let me also emphases that Eq. (13) provides no information about the relative sign of states $|2\rangle$ and $|1\rangle$. This depends on number of exchanges between anticommuting fields.

Commutation between b_j and f_j is a generic property, when one of two particles is bosonic:

$$[b_i, f_j] = [b_i^\dagger, f_j^\dagger] = \ldots = 0 \ . \tag{14}$$

The conservation of the charge Eq. (6) can be proven now $[\mathcal{H}, \mathcal{Q}] = 0$ by making use of the commutation rules Eqs. (9,14).

The permutation of particles with antiparticles to be called swap. It can be defined as operator

$$\hat{S}_{ij} b_i f_j = f_j b_i \qquad \hat{S}\hat{I}_4 = \hat{I}_4 \hat{S} \ , \tag{15}$$

and it commutes with the CPT transformation as follows from Eq. (14). The CPT invariance of ordered products is now formulated as

$$|2\rangle = \hat{S}^m \hat{E}^n |1\rangle \quad \Rightarrow \quad \hat{I}_4 |2\rangle = (-1)^n \hat{I}_4 |1\rangle \ . \tag{16}$$

It means that the relative sign of ordered products post CPT transformation is changed if and only if these states are related by the odd number of exchanges and arbitrary number of swaps.

IV. NON-INVARIANCE OF NON-ORDERED PRODUCTS AND OF WICK THEOREM

The theorem Eq. (16) cannot be extended to non-ordered products like $b_i b_k^\dagger b_j$. Let's compare:

$$\begin{aligned} b_i b_k^\dagger b_j &= b_k^\dagger b_i b_j + \delta_{ik} b_j = b_i b_j b_k^\dagger - \delta_{kj} b_i \\ f_j^\dagger f_k f_i^\dagger &= -f_j^\dagger f_i^\dagger f_k + \delta_{ik} f_j^\dagger = -f_k f_j^\dagger f_i^\dagger + \delta_{kj} f_i^\dagger \ . \end{aligned}$$

Assume that the CPT transformation from $b_i b_k^\dagger b_j$ to $f_j^\dagger f_k f_i^\dagger$ preserves the sign, meaning $\hat{I}_4 b_i b_k^\dagger b_j = f_j^\dagger f_k f_i^\dagger$. Then the transformation of $-\delta_{kj} b_i$ to $\delta_{kj} f_i^\dagger$ violates Eq. (1). If the CPT transformation from $b_i b_k^\dagger b_j$ to $f_j^\dagger f_k f_i^\dagger$ flips the sign $\hat{I}_4 b_i b_k^\dagger b_j = -f_j^\dagger f_k f_i^\dagger$ then the transformation of $\delta_{ik} b_j$ to $-\delta_{ik} f_j^\dagger$ violates Eq. (1).

Wick theorem as well as many body Green functions are not CPT invariant in present theory. The process with many indistinguishable particles is not the CPT mirror of the same process with many indistinguishable antiparticles. Comparing Wick theorem for four fields, Table III, we see the term $\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}$ in the right hand side of expression for $b_i b_j b_k^\dagger b_l^\dagger$. It cannot be the image of the term $\delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$ in the right hand side of expression for $f_i f_j f_k^\dagger f_l^\dagger$. The lead order diagrammatic expansion of two body Green function is given by above terms with product of δ-functions, therefore the evaluation of the two-body Green function for particles and antiparticles is not the same.

V. TWELVE POSTULATES FOR SUPERSYMMETRIC SPINOR FIELDS

Let's put together all rules required for consistent supersymmetric theory of spin 1/2 fields. The CPT transformation

(i) converts particle creation operators to antiparticle annihilation operators and vise versa;

(ii) converts normally ordered products of operators to normally ordered products of operators and antinormally ordered products of operators to antinormally ordered products of operators

(iii) converts anticommuting half-integer spin fields to commuting fields and vise versa, and therefore it anticommutes with exchange for half-integer spin fields;

(iv) keeps bosonic commutation rules for integer spin particles and antiparticles;

(v) converts valid commutation rules to valid commutation rules;

##			Wick theorem for four operators
#1	$\{f_j, f_k^\dagger\} = \delta_{jk}$	original:	$f_i f_j f_k^\dagger f_l^\dagger = f_l^\dagger f_k^\dagger f_j f_i - f_l^\dagger f_i \delta_{jk} - f_k^\dagger f_j \delta_{il} + f_k^\dagger f_i \delta_{jl} + f_l^\dagger f_j \delta_{ik} + \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$
	$\{b_j, b_k^\dagger\} = \delta_{jk}$	4-inverted:	$b_i^\dagger b_j^\dagger b_k b_l = b_l b_k b_j^\dagger b_i^\dagger - b_l b_i^\dagger \delta_{jk} - b_k b_j^\dagger \delta_{il} + b_k b_i^\dagger \delta_{jl} + b_l b_j^\dagger \delta_{ik} + \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$
#2	$\{f_j, f_k^\dagger\} = \delta_{jk}$	original:	$f_i f_j f_k^\dagger f_l^\dagger = f_l^\dagger f_k^\dagger f_j f_i - f_l^\dagger f_i \delta_{jk} - f_k^\dagger f_j \delta_{il} + f_k^\dagger f_i \delta_{jl} + f_l^\dagger f_j \delta_{ik} + \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$
	$\{b_j, b_k^\dagger\} = \delta_{jk}$	4-inverted:	$b_l b_k b_j^\dagger b_i^\dagger = b_i^\dagger b_j^\dagger b_k b_l - b_i^\dagger b_l \delta_{jk} - b_j^\dagger b_k \delta_{il} + b_i^\dagger b_k \delta_{jl} + b_j^\dagger b_l \delta_{ik} + \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$
#4	$\{f_j, f_k^\dagger\} = \delta_{jk}$	original 1:	$f_i f_j f_k^\dagger f_l^\dagger = f_l^\dagger f_k^\dagger f_j f_i - f_l^\dagger f_i \delta_{jk} - f_k^\dagger f_j \delta_{il} + f_k^\dagger f_i \delta_{jl} + f_l^\dagger f_j \delta_{ik} + \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$
	$[b_j, b_k^\dagger] = \delta_{jk}$	original 2:	$b_l b_k b_j^\dagger b_i^\dagger = b_i^\dagger b_j^\dagger b_k b_l + b_i^\dagger b_l \delta_{jk} + b_j^\dagger b_k \delta_{il} + b_i^\dagger b_k \delta_{jl} + b_j^\dagger b_l \delta_{ik} + \delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}$
		4-inverted 1:	$b_l b_k b_j^\dagger b_i^\dagger \neq b_i^\dagger b_j^\dagger b_k b_l + b_i^\dagger b_l \delta_{jk} + b_j^\dagger b_k \delta_{il} - b_i^\dagger b_k \delta_{jl} - b_j^\dagger b_l \delta_{ik} + \delta_{il}\delta_{jk} - \delta_{ik}\delta_{jl}$
		4-inverted 2:	$f_i f_j f_k^\dagger f_l^\dagger \neq f_l^\dagger f_k^\dagger f_j f_i - f_l^\dagger f_i \delta_{jk} - f_k^\dagger f_j \delta_{il} - f_k^\dagger f_i \delta_{jl} - f_l^\dagger f_j \delta_{ik} + \delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}$

TABLE III: The CPT transformation of Wick theorem for four operators. Wick theorem is not CPT invariant for the supersymmetric theory, variant #4, as opposite to the case, when all fields are anticommuting, variants #1 and #2.

(vi) conserves energy and inverts charge for single-occupied states;

(vii) inverts sign of one-body propagator;

(viii) converts multiple occupancy to zero occupancy;

(ix) violates rule of signs in Wick theorem, makes changes to many-body propagators and scattering amplitude of indistinguishable particles;

(x) prohibits decomposition of scalar (spin zero) and tensor fields (even spin particles) to product of half-integer spin quantized fields;

(xi) prohibits truly neutral half-integer spin particle which can be taken by the CPT transformation to itself;

(xii) acts as a scalar supercharge having no matrix representation in Fock space.

In connection with last postulate, the CPT transformation here plays a role of the scalar supercharge, because it converts bosons to fermions and vise versa. It is similar to the Q-matrix in the theory of disorder and chaos.[27] Coleman-Mandula theorem allows the scalar supercharge[28], however it was never considered before in supersymmetric many body theories, because of restrictions by Pauli exclusion principle.

As opposite to other supersymmetric theories, it is impossible to write the supercharge $\hat{I}_4$ in terms of particle fields, or as a matrix acting in Fock space, e.g. $\hat{I}_4 b = U^{-1} b U$. In both cases $\hat{I}_4$ will commute with the exchange and violate Eq. (4).

In addition to above 12 postulates, difficulties are foreseen for the path integral method. An integral over commuting variables should be used for a retarded path and an integral over anticommuting variables for an advanced path.

Comparing the today textbook QED with the supersymmetric QED, one concludes that crossections of single particle processes like Thomson scattering and electron positron annihilation should stay the same, because they have no channels with exchange of indistinguishable particles. The annihilation will produce pair of orthogonal photons, because the theory preserves internal symmetries of particles including parity.

VI. GROUND STATE OF POSITRONS IN AN ANTIATOM

The best approach for calculation of S-wave state of atoms with 1 and 2 electrons is the variational method of effective charge.[29][Problem to §69] For the ground state of Helium atom it gives 2% error.

We will apply the variational method of an effective charge to the antiatom; it should be valid under assumption that all Z positrons are condensed in lowest energy S-wave state. The antiatom Hamiltonian describes Z positrons in Coulomb field of nuclear of charge $-Z$. In atomic units it reads

$$\hat{H} = \sum_j \left[-\frac{1}{2}\nabla_j^2 - \frac{Z}{r_j} \right] + \frac{1}{2}\sum_{i\neq j} \frac{1}{|\vec{r}_i - \vec{r}_j|} \; . \tag{17}$$

The trial wave function for positron condensate in the ground state to be taken as

$$\psi = \prod_j \phi(r_j) \; , \quad \phi(r) = \sqrt{\frac{Z_{\rm eff}^3}{\pi}} e^{-Z_{\rm eff}\, r} \tag{18}$$

and the energy functional in Hartree approximation becomes

$$E = \langle\psi|\, \hat{H}\, |\psi\rangle = Z\frac{Z_{\rm eff}^2}{2} - Z^2 Z_{\rm eff} + \frac{5}{8}\frac{Z(Z-1)}{2} Z_{\rm eff} \tag{19}$$

It has minimum at

$$Z_{\rm eff} = \frac{11}{16} Z + \frac{5}{16} \tag{20}$$

and the ground state energy of the antiatom becomes

$$E_{\rm AA} = -\frac{1}{2}\frac{11^2}{16^2}(Z + 5/11)^2 Z \sim -0.24 Z^3 \; . \tag{21}$$

The bound state for positrons is possible but it is much more dense than the ground state of regular atom.

The most of positrons are located at $r \lesssim R_{\rm AA} = 1/Z_{\rm eff}$ meaning that the antiatoms are much smaller than atoms at large Z. The density at the origin $r = 0$ is $\rho_{\rm AA} \sim Z/(R_{\rm AA})^3 \sim Z^4$, where AA means antiatom. For comparison the ground state energy of the Thomas-Fermi model of atom with Z electrons is $E_{\mathrm{TF}} \sim -Z^{7/3}$, the radius $R_{\mathrm{TF}} \sim Z^{-1/3}$, and the density at the origin is $\rho_{\mathrm{TF}} \sim Z/(R_{\mathrm{AA}})^3 \sim Z^2$.

In the same approximation the wave function of the excited state is $\psi_1 = Z^{-1/2}\sum_i \phi_1(r_i)\prod_{j\neq i}\phi(r_j)$ where $\langle\psi_1|\,\psi_1\rangle = 1$ and $\langle\psi_1|\,\psi\rangle = 0$. The exponent Z'_{eff} in $\phi_1(r) \sim \exp(-Z'_{\mathrm{eff}}r)(1-(Z'_{\mathrm{eff}}+Z_{\mathrm{eff}})r/3)$ obtained by minimization of $E_1 = \langle\psi_1|\,\hat{H}\,|\psi_1\rangle$ turns out to be closed to 2, meaning that the wave function of the excited positron ϕ_1 is same as of $n = 2$ state of $Z = 2$ atom. The reason is the strong screening of the nuclear field by the condensate; factor 2 comes from symmetrization of the wave function. The excitation energy of a positron for large Z is therefore close to the ionization energy $\sim -\partial E_{\mathrm{AA}}/\partial Z \sim 0.7Z^2$.

The ground state energy of condensate of charged bosons should be lower than E_{AA} and can be found by Bogoliubov transformation.[30] Coulomb interaction creates plasma waves with wave number $k^2 \sim \rho/k^2$, where k^2 comes from kinetic term and ρ/k^2 comes from Coulomb repulsion between positrons. The plasma frequency is $\omega_p = \sqrt{4\pi\rho} \sim Z^2$ and wavenumber is $k_p \sim \rho^{1/4} \sim Z$. Energy of these waves contribute to the ground state energy $U_{AA} \sim -R_{AA}^3\int k^2 d\vec{k} \sim -R_{AA}^3\rho^{5/4} \sim -Z^2$. It is small compare to the energy of the zero state E_{AA} and need not be considered for stability of bound bosons.

Excitations of the condensate have gap $\sim \omega_p \sim Z^2$ comparable with the ionization energy. Therefore, there are three competing effects for light absorption with energy close to Z^2: antiatom ionization, excitation of one positron from $n = 1$, $Z_{\mathrm{eff}} \sim Z$ state to $n = 2$, $Z'_{\mathrm{eff}} \sim 2$ state, and excitation of positron condensate.

VII. SUMMARY

In summary we postulate the theory of commuting antimatter half integer spin fields. We calculated the CPT transformation for charge, energy, green function, ordered products, commutators and Wick theorem. All single-particles operators are CPT invariant; Wick theorem and interactions are not CPT invariant.

The theory predicts absence of the periodic table for antimatter. Positrons in an antiatom shell occupy only lowest S-state and form positron condensate. The condensate wave function is computed by the variational effective charge method, and the ground state energy was found $\sim -Z^3$. We identified few types of excitations, and all of them have energy $\sim Z^2$.

The maximal allowed Z for antiatoms depends on the limiting mechanism (which is still subject of controversy for atoms). For scenarios depending on charge density the antiatom maximal Z goes as square root of maximal Z for atoms.

It is not possible to build antimatter from antiatoms because of instability of bosonic matter and lack of the thermodynamic limit. Clusters of antimatter can possibly form bound states of high density. Indeed the antiatom shell dimension scales as Z^{-1} and therefore much smaller than size of atom $\sim Z^{-1/3}$, meaning collapse of antiatoms in absence of degeneracy pressure.

The work explains the baryon asymmetry of the universe (BAU). Amount of antimatter could possibly be equal to amount of matter, but it should exist in highly dense form and have very peculiar absorption spectra.

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