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Integration out Vector and Scalar fields upon SO(N,3N) to SO(1,3) lowering of spatial symmetry

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We study Maxwell Lagrangian upon raise to $SO(N,3N)$ and lowering to $SO(1,3)$ of the spatial symmetry. The action in $SO(N,3N)$ unifies electromagnetic and atomic forces of $SO(1,3)$ in one massive $SO(N,3N)$ vector.

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I. INTRODUCTION

In few previous publications the author put forward an idea of modern world being result of spatial symmetry lowering from $SO(N,3N)$ to $SO(1,3)$. This exercise was undertaken to make connection between exchange of spinor fields and spatial rotations in $SO(N,3N)$. Particularly I've reported anticommutation of left spinors and commutation of right spinors meaning that Dirac's Lagrangian must be supersymmetric.

This paper deals with integration out procedure for scalar and vector fields.

II. SYMMETRY LOWERING FOR SCALAR FIELD

The scalar Lagrangian in 4 and in $4N$ dimensions for the scalar field a looks the same

$$\mathcal{L} = \frac{1}{2}\left[(\partial a)^2 - (m_N)^2 a^2\right] \ , \tag{1}$$

where m_N depends on N and for $N=1$ will be denoted as m_a. The N-scalable form of the action becomes

$$\begin{aligned} i\mathcal{S} &= i\int d^4x\mathcal{L} &(2)\\ i\mathcal{S}_N &= \frac{i}{\lambda^{N-1}N}\int d^{4N}x\mathcal{L}\ . &(3)\end{aligned}$$

where Eq. (3) becomes Eq. (2) for $N=1$. The action here is written in natural units $\hbar = c = 4\pi\epsilon_0 = 1$. In first case Eq. (2) the dimensionality of the field is $[\phi] = x^{-1}$, and in the second case Eq. (3) the dimensionality of the field is $[\phi] = x^{-N}$ leading to $[\lambda] = x^2$.

Upon the symmetry lowering from $SO(N,3N)$ to $SO(1,3)$ the branching of the field

$$a(x_1,\ldots,x_j) \to \prod_{j=1}^{N} a(x_j) \tag{4}$$

leads to new action with $m_a^2 = m_N^2/N$

$$i\mathcal{S}_N \to \frac{i}{\lambda^{N-1}}\int d^4x\frac{1}{2}\left[(\partial a)^2 - m_a^2a^2\right]\left[\int d^4xa^2\right]^{N-1} \tag{5}$$

We will call it equivalent to $i\mathcal{S}$ in rapid decay approximation upon the self-consistency condition

$$\lambda^{N-1} = \int D[a]e^{i\mathcal{S}}\left[\int d^4xa^2\right]^{N-1} \tag{6}$$

and the leading term is

$$\lambda = \int d^4xG(x,x), \quad G(x,y) = \int D[a]e^{i\mathcal{S}}a(x)a(y) \tag{7}$$

which is big as number of fields in the universe.

III. SYMMETRY LOWERING FOR MAXWELL LAGRANGIAN

The Maxwell Lagrangian and action for vector field A_μ in 4 dimensions and for $A_{\mu j}$ in $4N$ dimensions are written in scalable form as

$$\begin{aligned} \mathcal{L}_A &= \frac{1}{2}A_{\mu i}\left[(\partial^2 + m_{AN}^2)g_{ij}^{\mu\nu} - \partial_i^\mu\partial_j^\nu\right]A_{\nu j} &(8)\\ i\mathcal{S}_N &= \frac{i}{\lambda^{N-1}N}\int d^{4N}x\mathcal{L}_A\ . &(9)\end{aligned}$$

where at $N=1$ we expect $m_{AN}\to 0$.

The symmetry lowering for the vector field works as

$$A_{\mu j}(x_1,\ldots,x_j) \to \sum_{j=1}^{N} A_\mu(x_j)\prod_{k\neq j} a(x_k) \tag{10}$$

and symmetry lowering for the action is complicated expression

$$\begin{aligned} i\mathcal{S}_N \ \Rightarrow\ & \frac{i}{\lambda^{N-1}}\left[\int d^4xa^2\right]^{N-1} &(11)\\ \times\ & \frac{1}{2}\left[\int d^4xA_\mu[(\partial^2 + m_{AN}^2)g^{\mu\nu} - \partial^\mu\partial^\nu]A_\nu\right] \\ -\ & \frac{i(N-1)}{2\lambda^{N-1}}\left[\int d^4xa^2\right]^{N-2}\int d^4xA^2\int d^4x(\partial a)^2 \end{aligned}$$

We will call it equivalent in rapid decay approximation to sum of vector and scalar actions $i\mathcal{S}_A + i\mathcal{S}_a$ where $i\mathcal{S}_A$

is $i\mathcal{S}_N$ from Eq. (9) at $N = 1$. Self-consistency conditions for the rapid decay approximation are Eq. (6) and

$$m_A^2 = m_{AN}^2 - (N-1)m_a^2 = 0 \tag{12}$$

where by definition

$$m_a^2 \equiv \frac{1}{\lambda^{N-1}} \int D[a] e^{i\mathcal{S}_a} \left[\int d^4 x a^2 \right]^{N-2} \int d^4 x (\partial a)^2 \, . \tag{13}$$

The action for the scalar field $i\mathcal{S}_a$ is given by Eq. (5) with $N \to N-1$ upon the condition

$$\lambda = (N-1) \int d^4 x g^{\mu\nu} D_{\mu\nu}(x,x) \tag{14}$$

$$D_{\mu\nu}(x,y) = \int D[A] e^{i\mathcal{S}_A} A_\mu(x) A_\nu(y) \tag{15}$$

which is different from Eq. (6) by factor $N-1$.

The Eqs. (6) and (15) are of tremendous physical meaning. These integrals are proportional to the total number of degrees of freedom of scalars and vectors in the universe. The dimension of the original $SO(N,3N)$ world is not arbitrary but it is given by ratio of numbers of scalar fields to the number of vector fields

$$N - 1 = \frac{\int d^4 x G(x,x)}{\int d^4 x g^{\mu\nu} D_{\mu\nu}(x,x)} \tag{16}$$

We therefore conclude that if the universe has N vector degrees of freedom (photons) it must have N^2 scalar degrees of freedom, which are all kinds of massive bosons.

IV. SCALING OF THE ELECTRIC CHARGE

The branching rules for $SO(N,3N)$ spinors are relatively straightforward; they become direct product of $SO(1,3)$ spinors. The $4N$ dimensional vector current therefore behaves as

$$J_{\mu j} \to \sum_j J_\mu(x_j) \prod_{i \neq j} \rho(x_i) \tag{17}$$

and therefore dimensionality of the vector current scales as $[J_{\mu j}] = x^{-3N}$. Together with scaling of the vector potential $[A_{\mu j}] = x^{-N}$ we arrive at following symmetry lowering relation for the action

$$\begin{aligned} i\mathcal{S}_N &= \frac{i}{N!!\eth^{\frac{N-1}{2}}} \int d^{4N} x A_{\mu j} J_j^\mu \\ &\Rightarrow \frac{i}{N!!\eth^{\frac{N-1}{2}}} N \int d^4 x A_\mu J^\mu \left[\int d^4 x a \rho \right]^{N-1} \end{aligned} \tag{18}$$

where the charge scaling factor $[\eth] = x^0$ is dimensionless. In the rapid decay approximation

$$\begin{aligned} N!!\eth^{\frac{N-1}{2}} &= N \int D[a] D[\rho] e^{i\mathcal{S}} \left[\int d^4 x a \rho \right]^{N-1} \\ \eth &= \left[\int d^4 x \int d^4 y G(x,y) \rho(x) \rho(y) \right] \end{aligned} \tag{19}$$

The scaling of this integral yet to be computed; clearly $\int d^3 x \rho$ is the number of fermions in the universe. Besides $\int d^4 y G(x,y) \sim 1/m_a^2$.

The current current interaction in $4N$ dimensions is $\langle A(x) A(y) \rangle / \eth^2$ scales as $(\lambda/\eth)^{N-1}/((N-2)!!)^2$. Could be N independent at the end. Need to put efforts to calculate in details.

V. CONCLUSIONS

We computed $SO(N,3N)$ dimensional Maxwell Lagrangian and action, assuming that the symmetry lowering to $SO(1,3)$ will restore well known 4-dimansional form of Maxwell Lagrangian and action. We found that $SO(1,3)$ action must have scalar field intercating with the fermion density, like atomic force. The total number of scalar fields should scale as N^2 – square of the total number of photons N.

The beauty of the theory is unification of electromagnetic and atomic forces in one $SO(N,3N)$ dimensional massive vector field. The mass of this field is the mass of $SO(1,3)$ scalar times $\sqrt{N}$.