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Spontaneous symmetry lowering of the SO(N,3N) metric field interacting with massles spinor

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The symmetry broken solution (ground state) of the $SO(N,3N)$ field theory is calculated by non-perturbative methods (N-times repeated Hubbard–Stratonovich transformation). The high symmetry fields are $4N$ dimensional metrics tensor and 2^{2N} dimensional spinor. The low symmetry fields are sum of 4 dimensional metrics tensors and product of 4 dimensional super–symmetric spinors of the $SO(1,3)$. All the efforts are made to find the solution where each embedded $SO(1,3)$ subspace get its own scale (shrink or expansion) breaking the global $SO(N,3N)$ symmetry.

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I. DECOUPLING WITH FEW ANTICOMMUTING VARIABLES

I've used the Hubbard–Stratonovich transformation for decoupling of N fields has the form

$$\begin{aligned} e^{iabc} &= \int \frac{d\xi_1 d\eta_1}{2\pi} e^{i\xi_1 a - i\xi_1\eta_1 - 0|\xi_1|} \\ &\times \int \frac{d\xi_2 d\eta_2}{2\pi} e^{i\xi_2 b - i\xi_2\eta_2 - 0|\xi_2|} \\ &\times \int \frac{d\xi_3 d\eta_3}{2\pi} e^{i\xi_3 c - i\xi_3\eta_3 - 0|\xi_3|} e^{i\eta_1\eta_2\eta_3} \end{aligned}$$

and N^2 pairs of variables $\xi_{jk}\eta_{jk}$ were taken to decouple N products of N terms in Eq.(11).

Assume now that the a, ξ_1, η_1 and b, ξ_2, η_2 are all anticommuting variables. We get

$$\begin{aligned} e^{iabc} &= \int \frac{d\eta_1}{2\pi} \int \frac{d\eta_2}{2\pi} i(a-\eta_1) i(b-\eta_2) \\ &\times \int \frac{d\xi_3 d\eta_3}{2\pi} e^{i\xi_3 c - i\xi_3\eta_3 - 0|\xi_3|} e^{i\eta_1\eta_2\eta_3} \end{aligned}$$

next step

$$\begin{aligned} e^{iabc} &= i^2 \int \frac{d\xi_3 d\eta_3}{2\pi} e^{i\xi_3 c - i\xi_3\eta_3 - 0|\xi_3|} \int \frac{d\eta_1}{2\pi} \int \frac{d\eta_2}{2\pi} \\ &\times (1 + i\eta_1\eta_2\eta_3)(ab - \eta_1 b - \eta_2 a + \eta_1\eta_2) \\ &= i^2 \int \frac{d\xi_3 d\eta_3}{2\pi} e^{i\xi_3 c - i\xi_3\eta_3 - 0|\xi_3|} \frac{1}{(2\pi)^2} e^{i\eta_3 ab} \end{aligned}$$

So the right formula is

$$\begin{aligned} e^{iabc} &= \int \frac{d\xi_1 d\eta_1}{i} e^{i\xi_1 a - i\xi_1\eta_1 - 0|\xi_1|} \\ &\times \int \frac{d\xi_2 d\eta_2}{i} e^{i\xi_2 b - i\xi_2\eta_2 - 0|\xi_2|} \\ &\times \int \frac{d\xi_3 d\eta_3}{2\pi} e^{i\xi_3 c - i\xi_3\eta_3 - 0|\xi_3|} e^{i\eta_1\eta_2\eta_3} \end{aligned}$$

where c, ξ_3, η_3 are commuting and the rest are anticommuting varaibles.

Let me look on tracing of action in little simplified case of zero v; The first step before integration over super–spinors ψ is to change the summation order in Eq.(12):

$$S_{hs} = S_{\xi\eta} + S_{\xi\psi} \tag{1}$$

$$S_{\xi\eta} = \sum_{k=1}^{N} \prod_{j=1}^{N} \eta_{jk} - \sum_{j,k=1}^{N} \xi_{jk}\eta_{jk} \tag{2}$$

$$\begin{aligned} S_{\xi\psi} &= \sum_{j=1}^{N} \int d^4x \psi_j^\dagger \gamma^0 \Big\{ \sum_{k=1}^{j-1} \xi_{jk} \gamma^{\text{FIVE}} \\ &+ \xi_{jj}\gamma^\mu i\partial_\mu + \sum_{k=j+1}^{N} \xi_{jk} \Big\} \psi_j \end{aligned} \tag{3}$$

The integration over $\psi^\dagger\psi$ produce the superdeterminant

$$\begin{aligned} e^{iS_\xi} &\equiv \int d\psi^\dagger d\psi e^{iS_{\xi\psi}} \\ &= \prod_{jp} \text{SDet} \Big\{ \sum_{k=1}^{j-1} \gamma^0\gamma^{\text{FIVE}} \xi_{jk} \\ &+ \xi_{jj}\gamma^0\gamma^\mu p_\mu + \sum_{k=j+1}^{N} \gamma^0 \xi_{jk} \Big\} \end{aligned} \tag{4}$$

where left–left and right–right are made from commuting variables and left-right and right-left are meade from anticommuting variables. The explicit from of the matrix in brackets for given momentum is

$$\begin{pmatrix} (p_0+p_z)\xi_{jj} & & A_{j>} - A_{j<} & \\ & (p_0+p_z)\xi_{jj} & & -A_{j>} - A_{j<} \\ A_{j>} + A_{j<} & & (p_0-p_z)\xi_{jj} & \\ & -A_{j>} + A_{j<} & & (p_0-p_z)\xi_{jj} \end{pmatrix}$$

The anticommuting channels are 2nd and 3rd rows and columns. Let's exchange 1st and 3rd row-columns to get

anticommuting channels in right place:

$$\begin{matrix}(p_0-p_z)\xi_{jj} & & A_{j>}+A_{j<} & \\ & (p_0+p_z)\xi_{jj} & & -A_{j>}-A_{j<} \\ A_{j>}-A_{j<} & & (p_0+p_z)\xi_{jj} & \\ & -A_{j>}+A_{j<} & & (p_0-p_z)\xi_{jj}\end{matrix}$$

So the Efetov's text book formula gives

$$\begin{aligned}&\mathrm{Sdet}\Big\{\Big\} = \left[\frac{p_0+p_z}{p_0-p_z} - 2\frac{A_{j>}A_{j<}}{(p_0-p_z)^2\xi_{jj}^2}\right] \\ &\times\left[\frac{p_0-p_z}{p_0+p_z} - 2\frac{A_{j>}A_{j<}}{(p_0+p_z)^2\xi_{jj}^2}\right] \\ &= 1 - 4\xi_{jj}^{-2}\frac{A_{j>}A_{j<}}{p_0^2-p_z^2} \qquad (5)\end{aligned}$$

which is the result of all cancellations in the superdeterminant. After all cancellations the relevant part of the action is

$$iS_\xi = -4\sum_{pj}\xi_{jj}^{-2}\frac{A_{j>}A_{j<}}{p_0^2-p_z^2}$$

Let me rewrite it as

$$iS_\xi = -i2\sum_j \chi_j \sum_{k'>j>k}\xi_{jk}\xi_{jk'}$$

and the remaining part of action is

$$S_{\xi\eta} = \sum_{k=1}^{N}\prod_{j=1}^{N}\eta_{jk} - \sum_{j,k=1}^{N}\xi_{jk}\eta_{jk}$$

Do it by hands, then search for matrix inversion.

$$\begin{aligned}&\int\frac{d\xi_1}{i}[1-i\chi(\xi_1+\xi_2)(\xi_4+\xi_5)](1-i\xi_1\eta_1) \\ &= -\eta_1 - \chi(1-i\xi_2\eta_1)(\xi_4+\xi_5)\end{aligned}$$

next one

$$\begin{aligned}&\int\frac{d\xi_2}{i}[-\eta_1-\chi(1-i\xi_2\eta_1)(\xi_4+\xi_5)](1-i\xi_2\eta_2) \\ &= -[-\eta_1\eta_2 - \chi\eta_2(\xi_4+\xi_5)] - \chi(-\eta_1)(\xi_4+\xi_5) \\ &= \eta_1\eta_2 + \chi(\eta_1+\eta_2)(\xi_4+\xi_5)\end{aligned}$$

Let's me do it otherwise

$$\begin{aligned}&\xi_1(\xi_3+\xi_4+\xi_5+\xi_6+\xi_7) \\ \to\ &\frac{1}{ii}(\eta_4\eta_5\eta_6\eta_7+\eta_3\eta_5\eta_6\eta_7+\ldots)\end{aligned}$$

II. CONFORMALLY FLAT METRIC FOR EINSTEIN-MAXWELL EQUATION

The conformally flat metric is chosen as

$$g_{\mu\nu} = e^{-u}\eta_{\mu\nu}$$

The Ricci tensor is

$$R_{\mu\nu} = u_{,\mu\nu} - \frac{1}{2}u_{,\mu}u_{,\nu} + \frac{1}{2}\eta_{\mu\nu}\eta^{\alpha\beta}(u_{,\alpha\beta}+u_{,\alpha}u_{,\beta})$$

and the factor R is

$$R = g^{\mu\nu}(3u_{,\mu\nu} + \frac{3}{2}u_{,\mu}u_{,\nu})$$

The action for Einstein-Maxwell equation is

$$S = \int d^4x\sqrt{g}\left[\frac{R}{4\pi^2k} + g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}\right]$$

III. NEW WAY

Let's look for the Einstein equation solution in the form

$$g_{j\mu j'\mu'} = \delta_{jj'}g^{(0)}_{\mu\mu'}v_j^2(t_j)$$

The Ricci curvature becomes

$$R = -6\sum\frac{\ddot{v}_j}{v_j^3}$$

and the left side of the Einstein equation in mixed indexes

$$-\frac{\dot{v}_j^2}{v_j^4} - \frac{\ddot{v}_j}{v_j^3} - R/2$$

IV. THE CONCEPT

The invariance upon Lorenz rotations of $SO(N,3N)$ fields (mainly $N=1$) is the starting point of all quantum field theories. The Lorenz invariance was challenged in many recent publications[1]; however it cannot be easily broken. The space metrics tensor (same as the gravitational field) can get the quantum observation value at the ground state, but this would mean that the space is curved. The Lorenz invariance will be preserved; just need to be modified for the curved space.

In few recent papers the Lorenz invariance is defined as "broken" if the light cone in the high symmetry state is broken to few small light cones in the low symmetry state. This typically occurs in Finsler geometries, and some other cases.[2–4]

This paper deals with relatively simple example of $SO(N,3N)$ to $SO(1,3)$ symmetry lowering. Here the big light cone get broken to small light cones

$$\sum_{j\mu}p_j^\mu p_{j\mu} = 0 \ \to\quad \forall j:\ \sum_\mu p_j^\mu p_{j\mu} = 0 \qquad (6)$$

where $j = 1\ldots N$ counts $SO(1,3)$ subspaces embedded into $SO(N,3N)$ and $\mu = 0\ldots 3$ counts axises of $SO(1,3)$.

The study of the $SO(N,3N)$ field theory is motivated by the possibility of the continuous particle exchange[5],

it becomes the regular exchange matrix once the symmetry is lowered down to $SO(1,3)$. For the $SO(N,3N)$ Lagrangian we will need the 2^{2N} dimensional spinor field $\psi^N_\alpha(x)$ and the symmetric traceless tensor field $h^{\mu'}_{j\mu j'}$.

In order to break the $SO(N,3N)$ symmetry the quantum field h should get the observation value in each of N embedded $SO(1,3)$ subspaces:

$$\langle |\, h^\mu_{j\mu j}\, | \rangle = v_j \ , \quad \mathrm{Tr}\, \langle |\, h\, | \rangle = 4 \sum v_j = 0 \ . \tag{7}$$

The values v_j will describe the shrink $v_j > 0$ or expansion $v_j < 0$ of j-th subspace; the metrics tensor for each subspace is $g_{jj} = \mathrm{diag}(1+v_j, -1-v_j, -1-v_j, -1-v_j)$, and $\sqrt{-\det(g_{jj})} = (1+v_j)^2$. Once each of N subspaces get its own scale, the $SO(N,3N)$ symmetry becomes broken. However the $SO(1,3)$ symmetry with-in each sub-space is preserved.

Strictly speaking we should write down the Einstein–Hilbert action for the field h.[6, 7] However we are looking for coordinate independent solutions; these solutions are either uniform shrinks or expansions. In any case the space curvature is zero, and the Lagrangian written in terms of the Ricci tensor will not describe the symmetry lowering.

In the next section the $SO(N,3N)$ invariant Lagrangian with the coordinate independent space metrics is explicitly written down; the metrics fields h interacts with the unsymmetrized energy–momentum tensor. Then we assume that the symmetry is broken in the low symmetry state; substitute the low symmetry wave function; decouple all fields by N-times repeated Hubbard–Stratonovich transformation; integrate out the superspinors and calculate the superdeterminants[8]; use the method of steepest descent and finally arrive to the set of non-linear algebraic equations.

In the low-symmetry state energy of spinor field can be non-uniformly distributed between subspaces and this should be accounted by the chemical potential and the temperature. However, the accounting of the non-zero temperature is postponed. There is long discussion in the literature if the supersymmetry can be preserved at non-zero temperature.[9]

Overall I was not able to find solution where some of subspaces get significant scale shrink or expansion due to condensation of the spinor energy. So the paper summarizes all done so far calculations.

V. THE HIGH AND LOW SYMMETRY ACTIONS.

The flat metrics h should be accounted directly in the action in a way similar to the random matrix theory and large N expansion methods in modern field theories

$$S = F(\frac{1}{2}\mathrm{Tr}h^2) + \int d^{4N}x\sqrt{-g}\mathcal{L}_N \ , \tag{8}$$

where $g = \det(g_{j\mu j'\mu'})$ and the $SO(N,3N)$ invariant Lagrangian[5] is

$$\mathcal{L}_N = i \sum \psi^{\dagger N} \gamma^{\mathrm{ZERO}} \gamma^\alpha_j \epsilon^\mu_{j\alpha j'} \partial_{j'\mu} \psi^N \tag{9}$$

where ϵ^μ_α is the vierbein, and refer[5] for the definition of γ-matrices. Last term is the chemical potential;

The branching rules for spinor and tensor fields upon $SO(1,3) \to SO(N,3N)$ embedding are simple: the spinor becomes the direct product of spinors; the tensor becomes sum of traceless symmetric tensors, sum of products of vectors and a scalar. We pick up only relevant fields:

$$h^{\mu'}_{j\mu j'} = v_j \delta_{jj'} \delta^{\mu'}_\mu \ , \quad \sum v_j = 0 \tag{10}$$

$$\begin{aligned} \mathcal{L}_N &= T \sum_{j\mu} \prod_{k=1}^{j-1} \psi^\dagger_k \gamma^0 \psi_k (1+v_k)^2 \\ &\times \ \psi^\dagger_j \gamma^0 \gamma^\mu i \partial_\mu \psi_j (1+v_j)^{3/2} \\ &\times \prod_{k=j+1}^{N} \psi^\dagger_k \gamma^0 \gamma^{\mathrm{FIVE}} \psi_k (1+v_k)^2 \ , \end{aligned} \tag{11}$$

where ψ is the $SO(1,3)$ super–spinor, only left components anticommutes.[5] The chosen form of $h^{\mu'}_{j\mu j'}$ describes the uniform compression $v_j < 0$ or expansion $v_j > 0$ of each of $j = 1 \ldots N$ subspaces; the vierbein factors are just $\epsilon^\mu_{j\alpha j'} = \delta_{jj'}\delta^\mu_\alpha (1+v_j)^{-1/2}$.

The righ hand side of Eq.(11) contains product of N spinors so the N-times Hubbard–Stratonovich transformation is required in order to decouple these fields. The result is

$$\begin{aligned} S_{hs} &= \sum_{j=1}^{N} \Bigg[\sum_{k=1}^{j-1} \xi_{kj} (1+v_k)^2 \int d^4x \psi^\dagger_k \gamma^0 \psi_k \\ &+ \ \xi_{jj} (1+v_j)^{3/2} \int d^4x \psi^\dagger_j \gamma^0 \gamma^\mu i \partial_\mu \psi_j \\ &+ \ \sum_{k=j+1}^{N} \xi_{kj} (1+v_k)^2 \int d^4x \psi^\dagger_k \gamma^0 \gamma^{\mathrm{FIVE}} \psi_k \\ &- \ \sum_{k=1}^{N} \xi_{kj} \eta_{kj} + \prod_{k=1}^{N} \eta_{kj} \Bigg] \end{aligned} \tag{12}$$

where we used $2N^2$ variables in order to decouple all the wave function products in the Lagrangian Eq.(11)

$$e^{i\int d^{4N}x \mathcal{L}_N} = \int \prod_{jk} \frac{d\xi_{jk} d\eta_{jk}}{2\pi} e^{iS_{hs}} \ . \tag{13}$$

Just for reference the Hubbard–Stratonovich transformation for decoupling of N fields has the form

$$\begin{aligned} e^{iabc} &= \int \frac{d\xi_1 d\eta_1}{2\pi} e^{i\xi_1 a - i\xi_1\eta_1 - 0|\xi_1|} \\ &\times \int \frac{d\xi_2 d\eta_2}{2\pi} e^{i\xi_2 b - i\xi_2\eta_2 - 0|\xi_2|} \\ &\times \int \frac{d\xi_3 d\eta_3}{2\pi} e^{i\xi_3 c - i\xi_3\eta_3 - 0|\xi_3|} e^{i\eta_1\eta_2\eta_3} \end{aligned}$$

and N^2 pairs of variables $\xi_{jk}\eta_{jk}$ were taken to decouple N products of N terms in Eq.(11).

VI. CALCULATION OF THE SUPERDETERMINANTS

The first step before integration over super–spinors ψ is to change the summation order in Eq.(12):

$$
\begin{aligned}
S_{hs} &= S_{\xi\eta} + S_{\xi\psi} + S_{\mu} && (14)\\
S_{\xi\eta} &= \sum_{k=1}^{N}\prod_{j=1}^{N}\eta_{jk} - \sum_{j,k=1}^{N}\xi_{jk}\eta_{jk} && (15)\\
S_{\xi\psi} &= \sum_{j=1}^{N}\int d^4x(1+v_j)^2\psi_j^\dagger\gamma^0\Big\{\sum_{k=1}^{j-1}\xi_{jk}\gamma^{\rm FIVE}\\
&+ \frac{\xi_{jj}}{\sqrt{1+v_j}}\gamma^\mu i\partial_\mu + \sum_{k=j+1}^{N}\xi_{jk}\Big\}\psi_j && (16)\\
S_{\mu} &= -\sum_{j=1}^{N}\mu_j\int d^4x(1+v_j)^2\psi_j^\dagger\psi_j && (17)
\end{aligned}
$$

where the chemical potential μ_j per subspace was introduced to control the number of particles. Otherwise there is no way to break the symmetry, we will have exactly one particle per subspace.

The integration over $\psi^\dagger\psi$ produce the superdeterminant

$$
\begin{aligned}
e^{iS_{v\xi}} &\equiv \int d\psi^\dagger d\psi e^{iS_{\xi\psi}+iS_\mu} && (18)\\
&= \prod_{jp}\mathrm{SDet}\Big\{(1+v_j)^2\Big[-\mu_j + \sum_{k=1}^{j-1}\gamma^0\gamma^{\rm FIVE}\xi_{jk}\\
&+ \frac{\xi_{jj}}{\sqrt{1+v_j}}\gamma^0\gamma^\mu p_\mu + \sum_{k=j+1}^{N}\gamma^0\xi_{jk}\Big]\Big\}^{-1}
\end{aligned}
$$

where left–left and right–right blocks should be taken as commuting variables and left-right and right-left blocks as anticommuting. The explicit from of the matrix in brackets for given momentum $p' = p\xi_{jj}(1+v_j)^{-1/2}$ is

$$
\begin{pmatrix}
p'_0-\mu_j+p'_z & & A_{j>}-A_{j<} & \\
& p'_0-\mu_j+p'_z & & -A_{j>}-A_{j<}\\
A_{j>}+A_{j<} & & p'_0-\mu_j-p'_z & \\
& -A_{j>}+A_{j<} & & p'_0-\mu_j-p'_z
\end{pmatrix}
$$

The anticommuting channels are 2nd and 3rd rows and columns. For the superdeterminant we stay with following matrix, where the anticommuting channels are in the bottom right:

$$
\begin{aligned}
\mathrm{Sdet}\Big\{\Big\} &= \left[\frac{p'_0-\mu_j+p'_z}{p'_0-\mu_j-p'_z} - \frac{A^2_{j>}-A^2_{j<}}{(p'_0-\mu_j-p'_z)^2}\right]\\
&\times\left[\frac{p'_0-\mu_j-p'_z}{p'_0-\mu_j+p'_z} - \frac{A^2_{j>}-A^2_{j<}}{(p'_0-\mu_j+p'_z)^2}\right]\\
&= \left[1-\frac{A^2_{j>}-A^2_{j<}}{(p'_0-\mu_j)^2-(p'_z)^2}\right]^2 \qquad (19)
\end{aligned}
$$

which is the result of all cancellations in the superdeterminant.

In the next section we will use the steepest descent method for the calculation of extremum of the action. In the vicinity of the extremum

$$
\begin{aligned}
iS_{v\xi} &= -2\sum_{jp}\frac{(1+v_j)^2}{\xi^4_{jj}}\ln\left[1-\frac{A^2_{j>}-A^2_{j<}}{(p_0-\mu_j)^2-\vec{p}^2}\right]\\
&\approx 2\sum_{jp}\frac{(1+v_j)^2}{\xi^4_{jj}}\frac{(A^2_{j>}-A^2_{j<})}{(p_0-\mu_j)^2-\vec{p}^2} && (20)\\
A_{j<} &= \sum_{k=1}^{j-1}\xi_{jk} \quad A_{j>} = \sum_{k=j+1}^{N}\xi_{jk} && (21)
\end{aligned}
$$

where I've rescaled $p' = p\xi_{jj}(1+v_j)^{-1/2} \to p$ and $p_z \to \vec{p}$.

We regularize the pole by $p^2 \to p^2 + i0$, then the sum over momenta gives

$$
iS_{v\xi} = 2i\mathcal{V}\sum_j \mu_j^2(1+v_j)^2\frac{A^2_{j<}-A^2_{j>}}{\xi^4_{jj}} \qquad (22)
$$

and $\mathcal{V}$ is the constant proportional to the system volume. The factor $(1+v_j)^2$ describes the volume scaling with space shrink/expansion.

Collecting back together all the terms contributing to the action in the low symmetry state we get

$$
\begin{aligned}
S &= F(\sum v_j^2) + \sum_{k=1}^{N}\prod_{j=1}^{N}\eta_{jk} - \sum_{j,k=1}^{N}\xi_{jk}\eta_{jk}\\
&+ 2\mathcal{V}\sum_j \mu_j^2(1+v_j)^2\frac{A^2_{j<}-A^2_{j>}}{\xi^4_{jj}} && (23)
\end{aligned}
$$

and make the following variables change

$$
\begin{aligned}
\xi_{jk} &\to \xi_{jk}\xi^2_{jj} && (24a)\\
\eta_{jk} &\to \eta_{jk}/\xi^2_{jj} && (24b)\\
\eta_{jj} &\to \eta_{jj}\xi^{2N-2}_{jj} && (24c)\\
\xi^{2N-1}_{jj} &\to \xi_{jj} && (24d)
\end{aligned}
$$

and the action becomes

$$
\begin{aligned}
S &= F(\sum v_j^2) + \sum_{k=1}^{N}\prod_{j=1}^{N}\eta_{jk} - \sum_{j,k}\xi_{jk}\eta_{jk}\\
&+ 2\mathcal{V}\sum_j \mu_j^2(1+v_j)^2\left(A^2_{j<}-A^2_{j>}\right) && (25)
\end{aligned}
$$

and unexpectedly all terms except $F(\sum v_j^2)$ collapse. Indeed, ξ_{jj} can be integrated out, leading to $\eta_{jj}=0$; consequently η_{jk} can be integrated out leading to $\xi_{jk}=0$ so

$$\int d\xi d\eta e^{iS} = e^{iF(\sum v_j^2)} \tag{26}$$

Therefore the ln in Eq. (20) must be preserved.

VII. THE STEEPEST DESCENT METHOD

There are two main approaches to find the suddle point of the action preserved in the form

$$\begin{aligned} S &= F(\sum v_j^2) + \sum_{k=1}^{N}\prod_{j=1}^{N}\eta_{jk} - \sum_{j,k=1}^{N}\xi_{jk}\eta_{jk} \\ &+ 2i\sum_{jp}\frac{(1+v_j)^2}{\xi_{jj}^4}\ln\left[1-\frac{A_{j>}^2-A_{j<}^2}{(p_0-\mu_j)^2-\vec{p}^2+i0}\right] \end{aligned} \tag{27}$$

The simple one is to adopt the ln expansion and stay with

$$\frac{\partial}{\partial v_j}F(\sum v_j^2)=0 \tag{28}$$

that definitely has solution $v_j=0$ and might have other solutions $v_j=\pm v\neq 0$. If the second minimum exist, then the symmetry can be broken spontaneously if some of $v_j=0$ and some other $v_j=\pm v$.

The more complicated approach is to find the true saddle point of entire system of metrics field h and the material field ψ. Taking derivatives with respect to v_j,ξ_{jk},η_{jk} the extremum location is at

$$\begin{aligned} \eta_{jk} &= 2i\mathcal{V}\sum_p \frac{\mu_j^2(1+v_j)^2}{\xi_{jj}^4}\begin{cases} A_{j<} & k<j \\ -A_{j>} & k>j \end{cases} \\ &\times \frac{1}{(p_0-\mu_j)^2-\vec{p}^2-A_{j>}^2+A_{j<}^2+i0} \end{aligned} \tag{29a}$$

$$\eta_{jj} = -8i\mathcal{V}\sum_p \frac{\mu_j^2(1+v_j)^2}{\xi_{jj}^5}\ln\Big[\cdots\Big] \tag{29b}$$

$$\xi_{jk} = \prod_{j'\neq j}\eta_{j'k} \tag{29c}$$

$$F'v_j = -4i\mathcal{V}\sum_p \frac{\mu_j^2(1+v_j)}{\xi_{jj}^4}\ln\Big[\cdots\Big] \tag{29d}$$

that is power N non-linear equation with $2N^2+N$ variables.

VIII. CONCLUSIONS

The solution of the saddle point equations is currently beyond my bandwidth. I should try the renorm-group analysis to see if the accumulation of the energy in of the sub-spaces can come with scale shrink or expansion which in turn will drive more energy to come into this subspace.

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