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Twelve postulates for a supersymmetric world

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Pauli exclusion principle might not work for antimatter half-integer spin fields. A consistent theory is postulated. We touch briefly on the experimental verification of antimatter statistics. The theory, if valid, would explain the baryon asymmetry of the universe. The quantization of gravity will not be possible. A Majorana particle will not exist.

INTRODUCTION

This work is motivated by a growing interest in the physics community to the problem known as the baryon asymmetry of the universe (BAU), or the abundance of observable antimatter in the universe.[1] The sky map of gamma-rays created from the annihilation of electrons (511 KeV) and from the annihilation of protons indicates negligible density of overlapping matter and antimatter.[2, 3] The observed fraction of antimatter in cosmic rays is very small.[4]

The BAU challenges the theory of the Big Bang. The electro-magnetic energy released by the Big Bang can only produce an equivalent amount of matter and antimatter by a process called matter creation; it is the reverse to annihilation. Somehow, all antimatter has disappeared as the universe has evolved. This is the most obscure and hard paradox in modern physics for following reasons.

First, the experimental difficulty comes from the low quantity and density of antimatter available for research. At most, one can handle a few thousands of antimatter atoms in a trap for few hours.[5] This was enough to prove with high accuracy that an antiproton is stable and has the same mass and absolute value of charge as a proton.[6] The gravity force for antiprotons and protons was also found to be the same.[7] However, the interaction of antimatter with antimatter would only be seen in a setup with at least 10^{18} antiatoms, which makes the inter-atomic distances comparable with the scattering length. Experimentalists then expect to see antimolecular lines in spectra and other effects of antiatomic and antinuclear interactions.

Second, the theoretical challenge comes from the three Saharov conditions of BAU: (i) an initial small matter–antimatter asymmetry; (ii) non-equilibrium of the early universe; (iii) violation of C-invariance and CP-invariance.[8] A consistent theory of BAU should include three theories explaining all three Saharov conditions.

Another reason for so little experimental knowledge of antimatter is related to the 4-inversion of space (the inversion of time and three coordinates), also known as CPT transformation. Consider a quantum particle created at point A, propagated to point B and annihilated at B. CPT transformation mirrors this event as a quantum antiparticle created at point B, propagated to point A, and annihilated there. This leads to the conclusion that all that is known about the world is valid for the anti-world by virtue of CPT symmetry. Experimental results obtained for matter are assumed to be valid for antimatter without verification.

The next section formulates a consistent theory in which CPT symmetry does not work for a many-body field; it works only for a single particle, as described above. We consider the case when CPT transformation changes sign in commutation relations. Here, matter and antimatter belong to opposite symmetry classes relative to particle permutations.

This helps to resolve the BAU paradox, because bosonic antimatter would not be supported by the degeneracy pressure and would collapse. There would be no antiatoms (except antihydrogen), no antiplanets and so on.[9, 10]

Pauli wrote in his Nobel Prize lecture[11] that an ensemble of several like particles that is a mixture of all symmetry classes is never realized in Nature. Pauli derived his conclusion from general problems of the statistical theory of heat. We note that particles and antiparticles are clearly distinguishable, and therefore it will be no issue if, for example, the specific heat of matter is different from the specific heat of antimatter under similar conditions.

Extra caution is required for theory that challenges well established principles such as Pauli theorem on the connection between spin and statistics. Therefore, this theory is formulated in terms of twelve postulates touching most aspects of the spin-statistics theorem. These postulates are not fully independent; however, it is hard to define which of them is less fundamental and can be stated a theorem.

CPT TRANSFORMATION POSTULATES

The twelve postulates of CPT transformation are as follows.

(i) CPT transformation converts particle creation operators to antiparticle annihilation operators and vice versa. This is a fundamental concept explained in all textbooks on quantum theory.[12, 13] CPT transformation contains charge conjugation (C in CPT transformation) and therefore transforms matter into antimatter. This postulate is common for half-integer and integer spin fields.

(ii) CPT transformation converts normally ordered products of operators to normally ordered products of operators and converts antinormally ordered products of operators to antinormally ordered products of operators. This postulate just means that the time inversion (T in CPT transformation) changes the order of operators. Consider the normally ordered product of operators, where all creation operators are to the left of all annihilation operators. The charge conjugation converts all creation operators to annihilation operators so the normal order becomes the antinormal order. The time inversion then takes the antinormal order back to the normal order.

(iii) CPT transformation converts anticommuting half-integer spin fields into commuting half-integer spin fields and vice versa. CPT transformation therefore anticommutes with exchange for half-integer spin fields. This is a new postulate. It also means that CPT transformation changes the sign of a product of operators depending on the number of permutations between half-integer spin fields.

(iv) CPT transformation keeps bosonic commutation rules for integer spin particles and antiparticles. Spin s particles can be regarded as composite fields made from $2s$ spin 1/2 particles.[13, §26] We therefore have inconsistency here; for example, some spin 1 field can be regarded as a composite field made from spin 1/2 bosons and spin 1/2 fermions. Presumably, this field has fermionic commutation rules in contradiction with postulated above. The Lorenz symmetry prohibits pairing of matter and antimatter conjugated fields in $SO(1;3)$ vector decomposition. Matter fields must be paired with matter conjugated fields and antimatter fields must be paired with antimatter conjugated fields. Therefore, the decomposition of vectors does not lead to any contradiction in the theory. The following postulate takes care of other tensor fields.

(v) The theory allows only spinor and vector quantum fields. Other tensor fields forms no algebra and cannot be quantized. Scalar and adjoint fields change symmetry upon permutation and therefore they don't form any algebra and cannot be quantized, where the calculation used decomposition to a product of two spinor fields.[14] A scalar field can be decomposed to a scalar product of two $SO(1;3)$ vector fields which is in turn product of four spinor fields. Four spinor fields cannot coexist at the same location and therefore do not help to define algebra for scalar fields. We meet same issue for higher rank tensors. Particularly the gravity cannot be quantized, because it is the symmetrized product of two $SO(1;3)$ vector fields which is in turn product of four spinor fields.

(vi) CPT invariance prohibits a truly neutral half-integer spin particle that can be converted by CPT transformation to itself. A Majorana particle is prohibited too because it is its own antiparticle.

(vii) CPT transformation converts valid commutation rules to valid commutation rules. Half-integer spin fields commuting to a delta-function become half-integer spin fields anticommuting to a delta-function. Integer spin fields commuting to a delta-function stay integer spin fields commuting to delta-function. In any case, the delta-function is invariant under CPT transformation. This statement about delta-function is not unique to the theory.

(viii) CPT transformation conserves energy and inverts the charge for single-occupied states. Explicit expressions for energy and charge are invariant under CPT transformation; however, the charge should be proportional to the difference between the number of particles and the number of antiparticles. For integer spin fields, this postulate requires field commutation, and this is aligned with postulates (i), (ii), and (iv). For half integer spins, the invariance of textbook expressions for energy and charge requires anticommuatation of negative frequency fields.[12] The statistics of positive frequency fields can be chosen to be bosonic, as in the present theory. CPT invariance of energy and charge can be proven straightforwardly by making use of of postulates (i), (ii), and (iii). The proposed theory requires a change in another well established convention: negative frequency fields should be recognized as particles and positive frequency fields should be recognized as antiparticles.

(ix) CPT transformation inverts the sign of a one-body propagator for both integer spin fields and half integer spin fields. This is a fundamental property of a one-body propagator that follows from the commutation of quantum fields out of the light cone.[15] Little math is needed to verify this postulate for the present theory by making use of postulates (i)-(iv). The vacuum observation value should be taken after CPT transformation. This postulate is the key foundation of Pauli spin-statistics theorem[16, 17] and it is fulfilled for supersymmetric theory proposed here.

(x) CPT transformation converts multiple occupancy to zero occupancy for half integer spin commuting states. Nothing can be done about this. Energy and charge are lost by CPT transformation of these states. CPT transformation is a hypothetical transformation, so nothing is lost in the real world. A similar problem arises for all supersymmetric theories at finite temperatures. The super-rotation mixes commuting and anticommuting fields, and they cannot satisfy Matsubara periodic and antiperiodic boundary conditions simultaneously. Some authors conclude that supersymmetry is broken by finite temperature.[18] Gibbs free energy and other thermodynamic potentials are not CPT invariant for half integer spin particles.

(xi) Wick theorem is not CPT-invariant. This is merely a consequence of postulates (iii) and (vii); however, it is kept as a postulate because of its high importance for the diagram technique. One should calculate the vacuum observation value of a product of four or more quantum fields before and after CPT transformation in order to understand this issue. Some δ-function terms will have the opposite sign before and after CPT transformation, implying the non-invariance of many-body propagators and scattering amplitudes of indistinguishable particles. The interaction between indistinguishable particles is therefore required to manifest the statistics.

(xii) CPT transformation acts as a scalar supercharge that has no matrix representation in Fock space and changes Fock space vacuum for half integer spin fields. The first part of this postulate is valid because matrix transformation cannot change commutation rules. Regarding the second part: formally, the vacuum is CPT invariant, meaning that it does not care what kind of particles are absent. However a supersymmetric vacuum in the present theory is not CPT invariant because Fock space vacuum states prepared for anticommuting fields are not the same as states prepared for commuting fields. This postulate is connected with postulate (x), but is more fundamental. Note that the scalar supercharge is allowed by Coleman-Mandula theorem[19, 20]

SUMMARY AND CALL FOR EXPERIMENT

The idea of supersymmetric spinor fields originates from particle exchange by making rotations in $SO(N, 3N)$ space.[14] The reported exchange matrix (analog of Heisenberg exchange operator) has a negative sign for permutation of the left spinors only; all other permutations have a positive sign. This is only hints at the idea that matter and antimatter belong to opposite symmetry classes with respect to permutations. Indeed, both matter and antimatter fields are made from linear combinations of left and right spinors, and therefore the abovementioned exchange matrix cannot be applied straightforwardly to matter and antimatter.

Another open issue is the calculation of various current-current correlation loop diagrams, particularly anomalies. The diagram technique requires a sign change for all fermionic loops; clearly only matter loops should change sign in the present theory. The explicit calculation of anomalies in the supersymmetric framework is a work in progress and will be published elsewhere.

Postulate (xi) requires the observation of particle–particle interaction as the only possible way to discover the particle statistics. We therefore call on the experimental physics community to measure the positron–positron Moller scattering. The scattering amplitude at small scattering angles is dominated by Coulomb repulsion, but high angles should reveal the positron statistics. The experiment therefore requires high luminosity of positron beams and enhanced positron sources similar to that used in electron–positron colliders.[21] The experiment can be set in one of the electron–positron colliders by reversing the polarity of one of the storage rings. This is a formidable task, and it can be performed once the program of electron-positron collision measurements is finished.

In summary, a consistent supersymmetric quantum theory is postulated. It predicts an unstable antiworld and explains matter antimatter asymmetry.

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TABLE I: Examples of calculations mentioned in some of postulates.

Spinor Field	CPT-image	Vector Field	CPT-image	postulates
c_p	$c^\dagger_{-p}$	a_p	$a^\dagger_{-p}$	(i)
c_{-p}	$c^\dagger_p$	a_{-p}	$a^\dagger_p$	(i)
$c_p c^\dagger_q - c^\dagger_q c_p = \delta_{\vec{p}\vec{q}}$	$c_{-p} c^\dagger_{-q} + c^\dagger_{-q} c_{-p} = \delta_{\vec{p}\vec{q}}$	$a_p a^\dagger_q - a^\dagger_q a_p = \delta_{\vec{p}\vec{q}}$	$a_{-p} a^\dagger_{-q} - a^\dagger_{-q} a_{-p} = \delta_{\vec{p}\vec{q}}$	(ii), (iii), (vii)
$\mathcal{E} = \sum_{\vec{p}} p_0 (c^\dagger_p c_p - c_{-p} c^\dagger_{-p})$ $\mathcal{Q} = \sum_{\vec{p}} (c^\dagger_p c_p + c_{-p} c^\dagger_{-p})$	$-\sum_{\vec{p}} p_0 (-c^\dagger_{-p} c_{-p} - c_p c^\dagger_p)$ $\sum_{\vec{p}} (-c^\dagger_{-p} c_{-p} + c_p c^\dagger_p)$	$\sum_{\vec{p}} p_0 (a^\dagger_p a_p + a_{-p} a^\dagger_{-p})$ $\sum_{\vec{p}} (a^\dagger_p a_p - a_{-p} a^\dagger_{-p})$	$\sum_{\vec{p}} p_0 (a^\dagger_{-p} a_{-p} + a_p a^\dagger_p)$ $-\sum_{\vec{p}} (a^\dagger_{-p} a_{-p} - a^\dagger_p a_p)$	(viii)
$\mathcal{G}(t) = \left\langle e^{-ip_0 t} c_p c^\dagger_p \right\rangle_{t>0}$ $+\left\langle e^{ip_0 t} c^\dagger_{-p} c_{-p} \right\rangle_{t>0}$ $-\left\langle e^{-ip_0 t} c^\dagger_p c_p \right\rangle_{t<0}$ $-\left\langle e^{ip_0 t} c_{-p} c^\dagger_{-p} \right\rangle_{t<0}$ $= e^{-ip_0 t}\big\|_{t>0} - e^{ip_0 t}\big\|_{t<0}$	$\left\langle e^{ip_0 t} c_{-p} c^\dagger_{-p} \right\rangle_{t<0}$ $-\left\langle e^{-ip_0 t} c^\dagger_p c_p \right\rangle_{t<0}$ $+\left\langle e^{ip_0 t} c^\dagger_{-p} c_{-p} \right\rangle_{t>0}$ $-\left\langle e^{-ip_0 t} c_p c^\dagger_p \right\rangle_{t>0}$ $= e^{ip_0 t}\big\|_{t<0} - e^{-ip_0 t}\big\|_{t>0}$	$\left\langle e^{-ip_0 t} a_p a^\dagger_p \right\rangle_{t>0}$ $+\left\langle e^{ip_0 t} a^\dagger_{-p} a_{-p} \right\rangle_{t>0}$ $-\left\langle e^{-ip_0 t} a^\dagger_p a_p \right\rangle_{t<0}$ $-\left\langle e^{ip_0 t} a_{-p} a^\dagger_{-p} \right\rangle_{t<0}$ $= e^{-ip_0 t}\big\|_{t>0} - e^{ip_0 t}\big\|_{t<0}$	$\left\langle e^{ip_0 t} a_{-p} a^\dagger_{-p} \right\rangle_{t<0}$ $+\left\langle e^{-ip_0 t} a^\dagger_p a_p \right\rangle_{t<0}$ $-\left\langle e^{ip_0 t} a^\dagger_p a_p \right\rangle_{t>0}$ $-\left\langle e^{-ip_0 t} a_p a^\dagger_p \right\rangle_{t>0}$ $= e^{ip_0 t}\big\|_{t<0} - e^{-ip_0 t}\big\|_{t>0}$	(ix)
		$\epsilon^\mu a_p = (c_p \tau^\mu c^\dagger_p$ $\pm c_{-p} \tau^\mu c^\dagger_{-p})/\sqrt{2}$	$\epsilon^\mu a^\dagger_{-p} = (c_{-p} \tau^\mu c^\dagger_{-p}$ $\pm c_p \tau^\mu c^\dagger_p)/\sqrt{2}$	by Lorenz invariance