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The Supersymmetric QED with the scalar supercharge

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The supersymmetric QED Lagrangian with electrons and positrons having opposite charge, parity and statistics is investigated in details related to the $\hat{C}\hat{P}\hat{T}$ invariance and particle permutations. In the present theory the charge conjugation anticommutes with permutation of particles. The measurement of positron-positron scattering and the dipositronium ground state should prove or rule out the proposed symmetry; however the experimental evidences are not yet available.

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I. INTRODUCTION

The history of the supesymmetric action is exposed in few excellent textbooks.[1–4] The real supersymmetric particles are yet to be discovered; in all supersymmetric Lagrangians the known material fields are replicated, the replica fields should have opposite statistics and different spin.

The fundamental obstacle in the development of the supersymmetric theories come from the Pauly principle making connection between group representation index (spin) and statistics.[5, 6] The many body supersymmetric action by definition combines material fields having opposite statistics, and by virtue of the Pauli principle, these fields cannot belong to the same representation. The only exception is the non-interacting single-body Lagrangian[4], where fields of the opposite statistics are allowed to have the same spin.

The choice of the supercharge symmetry is also restricted by the Coleman-Mandula theorem[2, 3], so the overall number of allowed supersymmetries is limited. The Coleman-Mandula theorem allows the scalar supercharge, unfortunately it is prohibited by the Pauly principle.

In the previous paper[7] the supersymmetric QED Lagrangian with the scalar supercharge was derived by the symmetry lowering of the single-body $4N$-dimensional action. In this way the spatial rotations in $4N$-dimensions became particle permutations in 4-dimensions. As it was reported, the left spinor fields must be anticommuting and the right fields must be commuting in order to comply with exchange rotations in $4N$-dimensions.

The quantization of the above field can be done in various ways; we want to preserve the high energy limit[1] where the left spinor becomes electron (in our case antiparticle) and the right spinor becomes positron (in our case particle). We will quantize the field rather straightforwardly by making use of particle $b^\dagger_{sp}, b_{sp}$ and antiparticle $f^\dagger_{sp}, f_{sp}$ creation and annihilation operators. The textbook expressions[8] for the energy and the charge will be precisely reproduced

$$\varepsilon \;=\; \sum_{sp} p_0 \left(b^\dagger_{sp} b_{sp} - f_{sp} f^\dagger_{sp} \right) \;, \qquad (1)$$

$$j_0 \;=\; \sum_{sp} \left(b^\dagger_{sp} b_{sp} + f_{sp} f^\dagger_{sp} \right) \;. \qquad (2)$$

The sign of the second term must be flipped to get the positively defined energy (and by similar argument) the correctly defined charge and the current. **Therefore it is enough to have only antiparticle fermionic field out of two spinors in order to get the positively defined energy and the correctly defined charge**.

None of the basic assumptions of the Pauli principle is relaxed in this theory: (i) - the Lorentz invariance, (ii) - the positively defined energy, (iii) - the positively normalized states, all are preserved. The Pauli principle has fourth (iv) implicit assumption of same statistics for particles and antiparticles. All proofs of the Pauli principle for spinors[1, 8] give the anticommutation of the antiparticles, the anticommutation of the particles follows from the (iv)-th assumption. In the present theory the the (iv)-th assumption is relaxed; particles are bosonic counterparts of the fermionic antiparticles; all connected by supersymmetric rotations with the scalar supercharge.

The QED kinetic term is always the mix of the left and right spinors by virtue of the Lorentz invariance. In present theory it becomes also mix of the commuting (bosonic) and anticommuting (fermionic) variables. Therefore, the kinetic term in the Lagrangian acquires the statistics flip operator, the scalar supercharge Q.

The major effort of this paper is to investigate the actions of $\hat{C}$, $\hat{P}$, and $\hat{T}$ transformations on the particle creation and annihilation operators, and on the plane waves. We expect the solution to be $\hat{C}\hat{P}\hat{T}$ invariant and having positively defined energy and correctly defined charge and current. This is indeed the case, therefore the statistics flip operator Q allows to bypass the Pauli principle.

The connection of the presented here theory to the reality yet to be verified. The fermionic fields can be identified with electrons, while bosonic field with positrons. The bosonic nature of positrons yet to be proved or ruled out.

The great interest is therefore raised to the systems having at least two interacting positrons. Unfortunately there is no experimental data neither on positron-positron scattering nor on the positronium molecule (Ps_2) ground state.

The electron-positron annihilation does not prove or disprove the present theory because it is described by the single line diagram, and there is nothing to be permuted. It is also known that electron and positron have opposite parity, however this follows from the Lorentz invariance (shown in the paper) and has nothing to do with the statistics of the particle fields.

The annihilation channel of the Bhabha scattering has two positron lines but they cannot be trivially permuted, because 1st positron is annihilated and 2nd positron is borne. The calculation in the Sec. B shows that this process is not sensitive to the positron statistics.

Notations in present paper are slightly different from previous one[7] because there is no need to go beyond 4 dimensions. So

$$\gamma^0 = \sigma_1 \otimes 1 , \quad \gamma^i = -i\sigma_2 \otimes \sigma_i \tag{3a}$$
$$\gamma^{\text{FIVE}} = -\sigma_3 \otimes 1 , \quad i\gamma^1\gamma^2 = 1 \otimes \sigma_3 \tag{3b}$$

where $\sigma_1, \sigma_2, \sigma_3$ are Pauli matrices. The indexing of the spinors is preserved in sense that

$$\psi_{L,R} = \frac{1 \mp \gamma^{\text{FIVE}}}{2}\psi , \quad \psi_{\uparrow,\downarrow} = \frac{1 \pm i\gamma^1\gamma^2}{2}\psi \tag{4}$$

where spin notations should be regarded only as index the component of the field operator.

II. THE FERMION-BOSON SWAP IN QED LAGRANGIAN

The theory start with supersymmetric QED Lagrangian as it was derived in [7]. Here the mass term is explicitly introduced in addition to the kinetic term derived by the symmetry lowering technique

$$\mathcal{L} = \psi^\dagger\gamma^0\gamma^\mu(i\partial_\mu + eA_\mu/c)\psi - m\psi^\dagger\gamma^0 Q\psi . \tag{5}$$

The fermion-boson swap operator Q acts as following

$$Q^\dagger = Q , \quad Q^2 = 1 , \tag{6}$$

and for this reason Q cannot be taken as Grassmannian variable. All bunch of the commutation relations become

$$\{\psi_L, \psi_L\} = \{\psi_L^\dagger, \psi_L^\dagger\} = 0 ,$$
$$[\psi_R, \psi_R] = [\psi_R^\dagger, \psi_R^\dagger] = 0 ,$$
$$[\psi_R, \psi_L] = [\psi_R^\dagger, \psi_L^\dagger] = 0 . \tag{7a}$$

The quantization of these fields is done in the standard way

$$\{\psi_{sL}(\vec r, t), \psi_{s'L}^\dagger(\vec r', t)\} = \delta_{ss'}\delta^3(\vec r - \vec r') \tag{7b}$$
$$[\psi_{sR}(\vec r, t), \psi_{s'R}^\dagger(\vec r', t)] = \delta_{ss'}\delta^3(\vec r - \vec r') . \tag{7c}$$

Upon the action of the supercharge Q commutation relations become opposite to above

$$[Q\psi_{sL}(\vec r, t), \psi_{s'L}^\dagger(\vec r', t)Q] = \delta_{ss'}\delta^3(\vec r - \vec r') \tag{8a}$$
$$\{Q\psi_{sR}(\vec r, t), \psi_{s'R}^\dagger(\vec r', t)Q\} = \delta_{ss'}\delta^3(\vec r - \vec r') , \tag{8b}$$

and combining Eqs. (7,8) one get

$$\{Q, \psi_{sL}(\vec r, t)\psi_{s'L}^\dagger(\vec r', t)\} = 2Q\delta_{ss'}\delta^3(\vec r - \vec r') \tag{9a}$$
$$\{Q, \psi_{sR}(\vec r, t)\psi_{s'R}^\dagger(\vec r', t)\} = 2Q\delta_{ss'}\delta^3(\vec r - \vec r') . \tag{9b}$$

However it is not clear to me if the operator Q can be taken as stand alone.

The fermionic QED can be recovered by changing right fields back to fermions

$$Q\psi_R \to \psi_R , \quad \psi_R^\dagger Q \to \psi_R^\dagger . \tag{10}$$

This transformation eliminates the supercharge Q from the Lagrangian Eq. (5) and all operators become anticommuting fermionic fields. However, once we introduced the supercharge Q we open the way to quantize the fields in terms of both fermionic and bosonic particles.

The interaction-free Lagrangian explicitly written in terms of the left and right waves is

$$\mathcal{L}_0 = \psi^\dagger \begin{pmatrix} \hat p_0 - \vec\sigma\hat{\vec p} & -mQ \\ -mQ & \hat p_0 + \vec\sigma\hat{\vec p} \end{pmatrix} \psi \tag{11}$$

the solutions are spin degenerate; we start with bosonic particles in volume $\mathcal{V}$, $p_0 = \sqrt{\vec p^2 + m^2}$

$$\psi = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\vec p}\frac{1}{\sqrt{2p_0}}\begin{pmatrix} Qb_p u_{Lp} e^{-ipx} \\ b_p u_{Rp} e^{-ipx} \end{pmatrix}$$
$$\psi^\dagger = \frac{1}{\sqrt{\mathcal{V}}}\sum_{\vec p}\frac{1}{\sqrt{2p_0}}\begin{pmatrix} b_p^\dagger Q u_{Lp}^\dagger e^{ipx}, & b_p^\dagger u_{Rp}^\dagger e^{ipx} \end{pmatrix} \tag{12}$$

$$u_{Lp} = \frac{p_0 + m + \vec\sigma\vec p}{\sqrt{2(p_0+m)}} , \quad u_{Rp} = \frac{p_0 + m - \vec\sigma\vec p}{\sqrt{2(p_0+m)}} .$$
$$u_{Lp}^2 + u_{Rp}^2 = 2p_0 , \quad u_{Lp}u_{Rp} = m$$
$$u_p = \begin{pmatrix} u_{Lp} \\ u_{Rp} \end{pmatrix} , \quad u_p^\dagger = \begin{pmatrix} u_{Lp}^\dagger, & u_{Rp}^\dagger \end{pmatrix}$$
$$u_p\gamma^0 u_p^\dagger = p_\mu\gamma^\mu + m \tag{13}$$

where the spin index and the summation over spins is assumed but it is not explicitly written. The amplitudes u_L and u_R have two rows each, thus making 4-component wave function ψ. Summations in Eq. (12) include the summation over columns of u_L and u_R and over rows of $u_L^\dagger$ and $u_R^\dagger$. This field carries the positive energy and the positive charge:

$$\varepsilon = \int d^3\vec r\, \psi^\dagger\hat p_0\psi = \sum_{\vec p} p_0 b_p^\dagger b_p > 0 \tag{14a}$$
$$n = \int d^3\vec r\, \psi^\dagger\psi = \sum_{\vec p} b_p^\dagger b_p > 0 \tag{14b}$$

and entire calculation only use $Q^2 = 1$.

The fermionic antiparticles are given by the 4-inversion of the above wavefunction

$$\begin{aligned}\hat{I}_4\psi(x) &= \gamma^{\text{FIVE}}\psi(-x) \\ &= \frac{1}{\sqrt{\mathcal{V}}}\sum_{\vec{p}}\frac{e^{ipx}}{\sqrt{2p_0}}\begin{pmatrix} -f_p^\dagger u_{Lp} \\ Qf_p^\dagger u_{Rp}\end{pmatrix} \\ \hat{I}_4\psi^\dagger(x) &= \frac{1}{\sqrt{\mathcal{V}}}\sum_{\vec{p}}\frac{e^{-ipx}}{\sqrt{2p_0}}\left(-f_p u_{Lp}^\dagger,\ f_pQu_{Rp}^\dagger\right), \quad (15)\end{aligned}$$

where summations include the sum over columns of u_L and u_R and over rows of $u_L^\dagger$ and $u_R^\dagger$. The action of the 4-inversion on the particle operators:

$$\begin{aligned}&\hat{I}_4 f_p^\dagger = Qb_p\ , \quad \hat{I}_4 f_p = b^\dagger Q \\ &\hat{I}_4 b_p = Qf_p^\dagger\ , \quad \hat{I}_4 b_p^\dagger = f_p Q\ . \quad (16)\end{aligned}$$

These formulas preserve the commutation relations of the left and the right spinors; they keep the annihilation operators with the positive "frequency" and the creation operators with the negative "frequency".

For the fermionic antiparticle the energy is positive and the charge is negative, as we expected from the very beginning, see Eq. (1),

$$\begin{aligned}\varepsilon &= \int d^3\vec{r}(\hat{I}_4\psi^\dagger)\hat{p}_0(\hat{I}_4\psi) \\ &= -\sum_{\vec{p}} p_0 f_p f_p^\dagger = \sum_{\vec{p}} p_0 f_p^\dagger f_p > 0 \quad (17a) \\ n &= \int d^3\vec{r}\hat{I}_4\psi^\dagger\psi = \int d^3 r(\hat{I}_4\psi^\dagger)(\hat{I}_4\psi) \\ &= \sum_{\vec{p}} f_p f_p^\dagger = -\sum_{\vec{p}} f_p^\dagger f_p < 0\ . \quad (17b)\end{aligned}$$

The energy of the system is positively defined, because it is positive for both particles and antiparticles. The general solution to the Lagrangian Eq. (11) is now written as

$$\psi \to \psi + \hat{I}_4\psi \quad (18)$$

which is automatically $\hat{C}\hat{P}\hat{T}$ invariant. In this way we arrived at the supersymmetric quantum field satisfying all conditions of the Pauli spin-statistics theorem.

The propagator for above waves has classical expression as shown in the Appendix Sec. A. It satisfies the equation of motion derived from the Lagrangian Eq. (11) without the operator Q; the operator Q is transferred to the definition of the propagator.

Let's track now the commutation relations between particle creation and annihilation operators. From Eqs. (7,8) we get

$$\begin{aligned}&[b_{sp}, b_{s'q}^\dagger] = \delta_{ss'}\delta_{pq}\ , \quad \{f_{sp}, f_{s'q}^\dagger\} = \delta_{ss'}\delta_{pq} \quad (19a) \\ &\{Qb_{sp}, b_{s'q}^\dagger Q\} = \delta_{ss'}\delta_{pq}\ ,\ [Qf_{sp}^\dagger, f_{s'q}Q] = \delta_{ss'}\delta_{pq} \quad (19b)\end{aligned}$$

and this is of course consistent with Eq. (16).

III. CONSISTENCY CHECK FOR THE OPPOSITE CHARGE, PARITY AND STATISTICS

It is known that particles and antiparticles in the regular QED have opposite charge and parity but the same statistics. In the present work particles and antiparticles have opposite statistics and the effect of the charge conjugation $\hat{C}$, space reflection $\hat{P}$ and the time reversion $\hat{T}$ is worth to investigate.

We will define the actions of the above transformations on the field operators in a standard textbook way

$$\begin{aligned}&\hat{P}\psi(t,\vec{r}) = \gamma^0\psi(t,-\vec{r})\ ,\ \hat{P}\psi^\dagger(t,\vec{r}) = \psi^\dagger(t,-\vec{r})\gamma^0 \\ &\hat{C}\psi(x) = \psi^\dagger(x)i\gamma^2\ , \quad \hat{C}\psi^\dagger(x) = i\gamma^2\psi(x) \\ &\hat{T}\psi(t,\vec{r}) = \psi^\dagger(-t,\vec{r})\gamma^1\gamma^3\ ,\ \hat{T}\psi^\dagger(t,\vec{r}) = \gamma^1\gamma^3\psi(-t,\vec{r}) \\ &\hat{I}_4\psi(x) \equiv \hat{C}\hat{P}\hat{T}\psi(x) = i\gamma^2\gamma^0\gamma^3\gamma^1\psi(-x) = \gamma^5\psi(-x)\end{aligned}$$

The only charge conjugation $\hat{C}$ is acting to convert particles to antiparticles (this can be seen by substitution the plane wave expansions into above formulas)

$$\hat{C}b_{sp} = f_{-sp}\ , \quad \hat{C}f_{sp} = b_{-sp}\ , \quad (21)$$

The space reflection change the statistics

$$\hat{P}b_{sp} = Qb_{sp}\ , \quad \hat{P}f_{sp} = Qf_{sp}\ . \quad (22)$$

In this way both the 4-inversion I_4 get all the properties Eq. (**??**). The general statement about Eq. (22) would be that **the charge conjugation takes the anticommuting world to the commuting antiworld**.

The permutation of two particles can be formally written as the action of the exchange operator

$$-f_pf_q = Ef_qf_p\ , \quad b_pb_q = Eb_qb_p \quad (23)$$

then the action of the Charge conjugation is

$$\begin{aligned}CEf_qf_p &= -b_pb_q = -ECf_qf_p \quad (24) \\ CEb_qb_p &= -f_pf_q = -ECb_qb_p \quad (25)\end{aligned}$$

and it anticommutes with the particle exchange, as was stated in the abstract.

IV. THE SUMMARY

In present theory particles and antiparticles have opposite charge, statistics and parity. The space reflection of the charge conjugated wave has opposite sign compare to the space reflection of the original wave[8]:

$$\hat{P}(\hat{C}\psi)\psi = -(\hat{C}\psi)\psi\ , \quad (26)$$

meaning the opposite parity of particles and antiparticles. This effect is seen experimentally as the orthogonal polarization of the annihilation photons [8]§88. This effect is independent of the particle statistics, and cannot rule out the present theory.

The single Q operator cannot stay in the scattering amplitude because it would imply odd number of particle and antiparticle operators; at the end some operators will not be able to find the pair. The even number of Q operators will give unity after the time ordering. Therefore the operator Q drops from diagrams computed with Lagrangian Eq. (5).

For the above reason we must preserve all QED diagram rules[8] with one exception: the permutation of particles preserve the sign (the permutation of antiparticles change the sign) of a diagram. The prediction for differential cross section of electrons and positrons becomes

$$d\sigma_{e^-e^-} < d\sigma_{e^-e^+} < d\sigma_{e^+e^+} \ , \quad \vec{p}^2 \ll m^2 \ , \quad (27a)$$
$$d\sigma_{e^+e^+} < d\sigma_{e^-e^+} < d\sigma_{e^-e^-} \ , \quad \vec{p}^2 \gg m^2 \ . \quad (27b)$$

Let me propose the experiment[9] with positron scattering on cold positron plasma[10], positronium[11] or trapped antihydrogen[12–14]. The angle dependence of the scattering cross section should satisfy Eq. (27)

The ground state spin of the positronium molecules[11], also known as dipositronium Ps2 would be $S = 1$ for commuting positrons. The non-zero Zeeman effect is predicted for Ps2 in subsequent publication[15], and it allows to identify the ground state spin.

Another prediction is the lack of the molecular antihydrogen, because for there is no exchange interaction for bosonic positrons. The hydrogen molecule binding energy ∼4.5eV is comparable with the binding energy of the hydrogen atom ∼13.6eV, so the molecular antihydrogen must be observed together with atomic antihydrogen. Surprisingly no world was said about molecular antihydrogen in few latest experiment reviews.[16, 17]

Let me mention at the end, that the world made from bosonic charges becomes unstable.[18] This explains, why antiworld cannot exist for long time, and never observed.

APPENDIX A: THE GREEN FUNCTION FOR MIXED FERMION-BOSON FIELD

The generic expression for the 4-current $j^\mu = \bar\psi Q\gamma^\mu\psi$ has left $j_L = \psi_L^\dagger\gamma^0\gamma^\mu\psi_L$ and right $j_R = \psi_R^\dagger\gamma^0\gamma^\mu\psi_R$ components made from fermionic and bosonic fields. However some diagrams might need to mix left and right fields, therefore the Green function should be defined with operator Q.

The Green function for positive time $t > 0$ describes the motion of particles, but this is only seen with the plane wave expansion Eqs. (12,13)

$$\begin{aligned} G(x) &= \begin{pmatrix} \langle|\,\psi_L(x)\psi_R^\dagger(0)Q\,|\rangle & \langle|\,\psi_L(x)\psi_L^\dagger(0)\,|\rangle \\ -\langle|\,\psi_R(x)\psi_R^\dagger(0)\,|\rangle & \langle|\,Q\psi_R(x)\psi_L^\dagger(0)\,|\rangle \end{pmatrix} \\ &= \begin{pmatrix} -\langle|\,Q\psi_L(x)\psi_R^\dagger(0)\,|\rangle & \langle|\,\psi_L(x)\psi_L^\dagger(0)\,|\rangle \\ -\langle|\,\psi_R(x)\psi_R^\dagger(0)\,|\rangle & -\langle|\,\psi_R(x)\psi_L^\dagger(0)Q\,|\rangle \end{pmatrix} \\ & \quad t > 0 \end{aligned} \tag{A1}$$

The Green function for negative time describe the motion of antiparticles, see Eqs. (15,18),

$$\begin{aligned} G(x) &= -\begin{pmatrix} \langle|\,\psi_R^\dagger(0)Q\psi_L(x)\,|\rangle & \langle|\,\psi_L^\dagger(0)\psi_L(x)\,|\rangle \\ \langle|\,\psi_R^\dagger(0)\psi_R(x)\,|\rangle & \langle|\,\psi_L^\dagger(0)Q\psi_R(x)\,|\rangle \end{pmatrix} \\ & \quad t < 0 \end{aligned} \tag{A2}$$

The Green function satisfies the equation of motion, which follows from Eqs. (5,7),

$$(i\partial_\mu\gamma^\mu - m)\,G(x) = \delta(x - x') \ , \tag{A3}$$

where the operator Q is moved from the Hamiltonian into the Green function.

APPENDIX B: RELATIVE SIGNS OF ELECTRON AND POSITRON SCATTERING DIAGRAMS

For the first glance the overall theory seems to be standard QED with particle annyhilation operator Qb_p. Let's check if the commutation rules for particles can be revealed in the real physical process.

The generic expression for the 4-current expanded over plane waves is

$$j^\mu(x) = \psi^\dagger\gamma^0\gamma^\mu\psi = j^\mu_{e^-} + j^\mu_{e^+} + j^\mu_{an} + j^\mu_{cr} \tag{B1a}$$

where one has the positron current

$$j^\mu_{e^+} = \sum_{\vec p\vec q} \frac{e^{ipx-iqx}}{2\sqrt{p_0q_0}} b_p^\dagger b_q u_p^\dagger\gamma^0\gamma^\mu u_q \tag{B1b}$$

the electron current

$$j^\mu_{e^-} = \sum_{\vec p\vec q} \frac{e^{-ipx+iqx}}{2\sqrt{p_0q_0}} f_p f_q^\dagger u_{-p}^\dagger\gamma^0\gamma^\mu u_{-q} \tag{B1c}$$

the pair creation current

$$j^\mu_{cr} = \sum_{\vec p\vec q} \frac{e^{ipx+iqx}}{2\sqrt{p_0q_0}} b_p^\dagger Q f_q^\dagger u_p^\dagger\gamma^0\gamma^\mu u_{-q} \tag{B1d}$$

and the pair annihilation current

$$j^\mu_{an} = \sum_{\vec p\vec q} \frac{e^{-ipx-iqx}}{2\sqrt{p_0q_0}} f_p Q b_q u_p^\dagger\gamma^0\gamma^\mu u_{-q} \ , \tag{B1e}$$

besides $u_{-p} = \gamma^{\rm FIVE}u_p$, similarly to the transformation rules of the plane waves. For two antiparticles $u_{-p}^\dagger\gamma^0\gamma^\mu u_{-q} = u_p^\dagger\gamma^0\gamma^\mu u_q$.

For the interaction of two particles we have following terms [8] §73 in the scattering matrix

$$\begin{aligned} S &= \frac{ie^2}{2!}\int d^4x d^4x' D_{\mu\nu}(x-x') Tj^\mu(x)j^\nu(x') \\ &= S^{\rm el}_{e^-e^-} + S^{\rm el}_{e^+e^+} + S^{\rm el}_{e^-e^+} + S^{\rm an}_{e^-e^+} \end{aligned} \tag{B2}$$

where $iD_{\mu\nu}(x-x')$ is the photon propagator, and T stands for the time ordering. We postpone taking matrix elements of $j^\mu(x)j^\nu(x')$ and transition to the scattering amplitude. For the Moller scattering

$$S^{\rm el}_{e^+e^+} + S^{\rm el}_{e^-e^-} = \frac{ie^2}{2!}\sum(u_p^\dagger\gamma^0\gamma^\mu u_q)(u_{p'}^\dagger\gamma^0\gamma^\nu u_{q'}) \times \frac{D_{\mu\nu,\ p-q=q'-p'}}{4\sqrt{p_0q_0p_0'q_0'}}\left(b_{q'}b_p^\dagger b_q b_{p'}^\dagger b_p^\dagger - f_{p'}f_p f_{q'}^\dagger f_q^\dagger\right)$$

the particle e^+e^+ channel is even upon particle permutations and the antiparticle e^-e^- channel is odd upon particle permutations, as well known for electrons. For the elastic part of the Bhabha scattering one has

$$S^{\rm el}_{e^-e^+} = \frac{ie^2}{2!}\sum(u_p^\dagger\gamma^0\gamma^\mu u_q)(u_{p'}^\dagger\gamma^0\gamma^\nu u_{q'}) \times \frac{D_{\mu\nu,\ p-q=p'-q'}}{4\sqrt{p_0q_0p_0'q_0'}}\left(b_q f_{p'} b_p^\dagger f_{q'}^\dagger + f_p b_{q'} f_q^\dagger b_{p'}^\dagger\right)$$

and for the annihilation channel one gets

$$S^{\rm an}_{e^-e^+} = -\frac{ie^2}{2!}\sum(u_p^\dagger\gamma^0\gamma^\mu u_{-q})(u_{p'}^\dagger\gamma^0\gamma^\nu u_{-q'}) \times \frac{D_{\mu\nu,\ p+q=p'+q'}}{4\sqrt{p_0q_0p_0'q_0'}}\left(f_{p'} b_{q'} b_p^\dagger f_q^\dagger + f_p b_q b_{p'}^\dagger f_{q'}^\dagger\right)$$

The scattering matrices $S^{\rm el}_{e^-e^-}$, $S^{\rm el}_{e^-e^+}$, and $S^{\rm an}_{e^-e^+}$ lead to well known scattering amplitudes for the Moller scattering of electrons and the Bhabha electron-positron scattering.

The positron-positron channel of the Moller scattering will have other sign of the interference term because the scattering matrix $S^{\rm el}_{e^+e^+}$ is made from all boson operators.

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